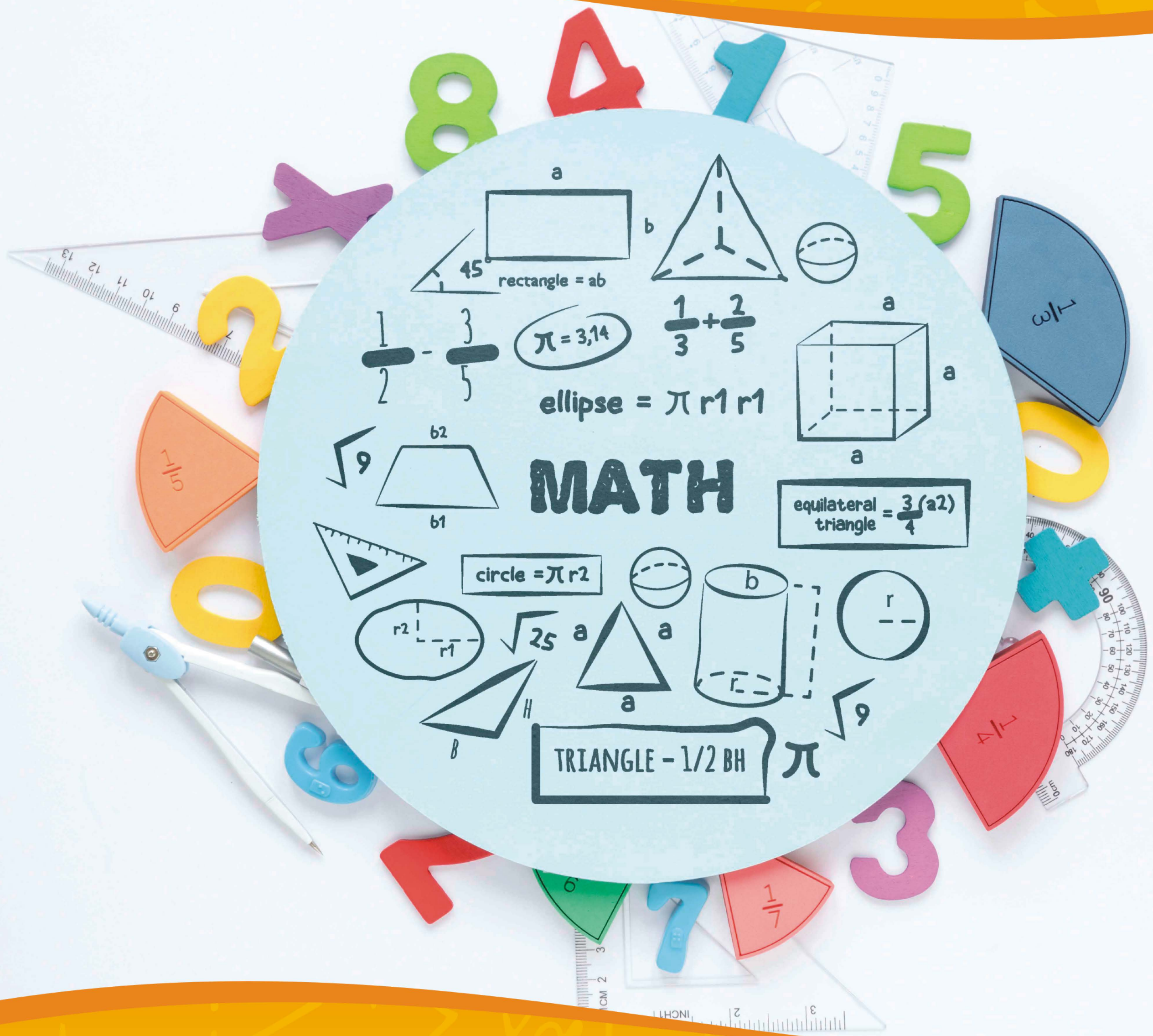
NEET Sarthi KOTA

SAMPLE MODULE

CLASS - 9th

MATHEMATICS



CHAPTER

NUMBER SYSTEM & POLYNOMIAL
SURDS AND INDICES

Online Platform for NEET, JEE & NTSE

NEET SARTHI PRE NURTURE MODULES DETAILS

SUBJECTS	Class 6	Class 7	Class 8	Class 9	Class 10
Physics	Module 1	Module 1	Module 1	Module 1	Module 1
Chemistry	Module 1	Module 1	Module 1	Module 1	Module 1
Biology	Module 1	Module 1	Module 1	Module 1	Module 1
Maths	Module 1	Module 1 Module 2	Module 1	Module 1 Module 2	Module 1 Module 2
Social Science	Module 1	Module 1	Module 1	Module 1 Module 2	Module 1 Module 2
Mental Ability	Module 1	Module 1	Module 1	Module 1	Module 1 Module 2
Total No. of Modules	6	7	6	8	9

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Mathematics

PRE-NURTURE DIVISION

Sample Module English

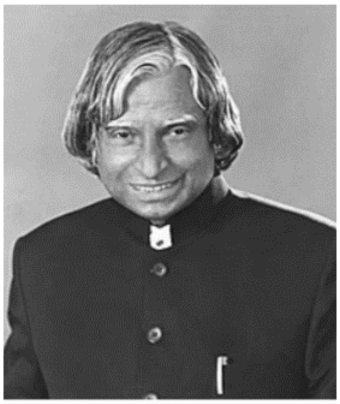
STUDENT NAME:________________________________

SECTION: ____________________ROLL NO: __________

CONTENTS

"To succeed in your mission,

you must have single-minded devotion to your goal."



Dr. A.P.J. Abdul Kalam

Dr. A.P.J. Abdul Kalam, popularly known as the 'Missile Man' of India, was a source of inspiration for tens and thousands of Indians. A league apart, his life philosophy and teachings are not only admired by the older generation, but especially reminisced by young. Kalam's prodigious rise from Rameswaram, a small but famous pilgrimage town in Tamil Nadu, led him to become one of the world's most accomplished leaders.

"All power is within you;

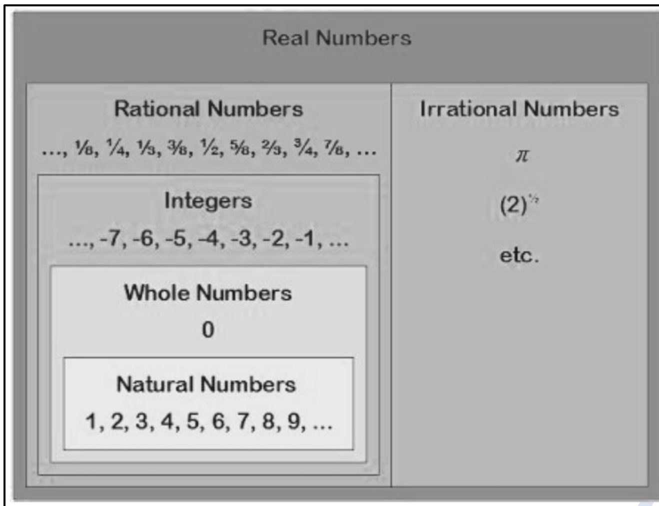
you can do anything and everything."

Swami Vivekananda

Swami Vivekananda ji's original name was Narendranath. He was born on 12th January, 1863 at Kolkata (Swamiji's Jayanti i.e. birth anniversary is celebrated as the 'International Youth Day'). Right from childhood, two aspects of his behavior could clearly be noticed. One was his devout and compassionate nature and the other was his readiness to perform any act of courage.

Chapter-01

Number System



Number System is not just a collection of numbers but it is a set together with rules such as addition, subtraction, multiplication, division etc.
" All rational & irrational numbers constitute real Numbers."

1.1 INTRODUCTION

In this chapter, we shall discuss types of numbers, laws of indices, surds, types of surds, laws of surds, the four basic operations on surds, their comparison etc. Basically, a surd is an irrational number. Hence, let us first review the types of numbers and recall their definitions.

- **Natural Numbers**

  All counting numbers are natural numbers. If N is the set of natural numbers, then N = {1, 2, 3, ...}

- **Whole Numbers**

  Natural numbers including zero represent the set of whole numbers. It is denoted by the symbol W.
  For Ex.: W = {0, 1, 2, 3....}

- **Integers**

  The set of integers, Z = {...., −3, −2, −1, 0, 1, 2, 3, ...}

- **Rational Numbers**

  Any number that can be expressed in the form p/q (where $q \neq 0$ and p, q are mutually coprime numbers) is called a rational number. All integers, all recurring decimals and all terminating decimals are rational numbers.

  Any integer can be expressed in the form $\frac{p}{q}$ $(q \neq 0)$.

Ex. $\frac{2}{1}, \frac{3}{1}, \frac{0}{1}$, etc. Any recurring decimal can be expressed in the form $\frac{p}{q}(q \neq 0)$.

Ex. $0.333... = \frac{1}{3}, 0.1666... = \frac{1}{6}$

Any terminating decimal can be expressed in the form $\frac{p}{q}(q \neq 0)$.

Ex. $0.5 = \frac{5}{10} \text{ or } \frac{1}{2}, 0.25 = \frac{25}{100} \text{ or } \frac{1}{4}$

- **Irrational Numbers**

  A number that cannot be expressed in the form $\frac{p}{q}$ (where $q \neq 0$), p and q are integers, is an irrational number or any non-terminating and non-recurring decimal is an irrational number.

  Ex. $\sqrt{2}, \sqrt{3}, \sqrt[3]{5}, \sqrt[4]{25}$ cannot be expressed in the form $\frac{p}{q}$ $(q \neq 0)$, where p and q are integers.

  $\therefore$ These numbers are irrational numbers.

- **Real Numbers**

  A number that is either rational or irrational is a real number.

- **Transcendental numbers**

  These are called so because they transcend the rules of Algebra. These are not algebraic in the sense that they defy addition, subtraction, multiplication and division and are not roots of any algebraic equation. All the same, transcendental numbers are as useful as any other real number. One such number is π which is defined as the ratio of the circumference to the diameter of any circle. As you may be aware, you need this number to calculate the areas of circles, surface areas and volumes of many standard solids.

  Thus the set of real numbers R consists of rational, irrational and transcendental numbers.

- **Number line**

  A straight line on which points are identified with real numbers is called a number line. Successive integers are placed on the number line at regular intervals. The following is an illustration of the number line.

- **Even numbers**

  All those numbers which are exactly divisible by 2 are called even numbers, e.g. 2, 6, 8, 10 etc. are even numbers.

- **Odd numbers**

  All those numbers which are not exactly divisible by 2 are called odd numbers, e.g. 1, 3, 5, 7 etc. are odd numbers.

- **Prime numbers**

  Except 1 each natural number which is divisible by only 1 and itself is called as prime number

  Ex. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, ... etc.

  - There are total 25 prime numbers upto 100.
  - There are total 46 prime numbers upto 200.
  - 2 is the only even prime number and the least prime number.
  - 1 is neither prime nor composite number.
  - There are infinite prime numbers.
  - A list of all prime numbers upto 100 is given below.

Table of prime Numbers (1-100):

2	11	23	31	41	53	61	71	83	97
3	13	29	37	43	59	67	73	89	
5	17	47	79	7	19				

Test to find whether a given number is a prime

- Select a least positive integer n such that n^2 > given number.
- Test the divisibility of given number by every prime number less than n.
- The given number is prime only if it is not divisible by any of these primes.

Ex.1: Investigate whether 571 is a prime number.

Sol. Since $(23)^2 = 529 < 571$ and $(24)^2 = 576 > 571$

$\therefore$ n = 24

Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19, 23. Since 24 is divisible by 2 but not by 5. So, 571 is not a prime number.

- **Co-prime**
  A pair of two natural numbers having no common factor, other than 1, is called a pair of co-primes.
  For Ex.: (3, 5), (4, 5), (5, 6), (7, 9), (6, 7) etc., are co-primes.
- **Twin primes**
  Prime numbers differing by two are called twin primes, e.g. (3, 5), (5, 7), (11, 13) etc., are called twin primes.
- **Prime triplet**
  A set of three consecutive primes differing by 2, such as (3, 5, 7) is called a prime triplet.
  "every prime number except 2 is odd but every odd number need not be prime."
- **Fractions**

(a)	Common fraction	:	Fractions whose denominator is not 10.
(b)	Decimal fraction	:	Fractions whose denominator is 10 or any power of 10.
(c)	Proper fraction	:	Numerator < Denominator i.e. . $\frac{3}{5}$
(d)	Improper fraction	:	Numerator > Denominator i.e. . $\frac{5}{3}$
(e)	Mixed fraction	:	Consists of integral as well as fractional part i.e. . $3\frac{2}{7}$
(f)	Compound fraction	:	Fraction whose number and denominator themselves are fractions i.e. . $\frac{2/3}{5/7}$

- **Composite Numbers**
  All natural numbers, which are not prime are composite numbers. If C is the set of composite number Then C = {4, 6, 8, 9, 10, 12.....}
- **Imaginary numbers**
  All the numbers whose square is negative are called imaginary numbers, Ex., 3i, –4i......; where i = $\sqrt{-1}$.

1.1.1 Inserting a number between two rational numbers

For determining one or more than one rational number between two given numbers, we use the following method.

- **When one rational Number is determined**

Let x and y be two numbers, such that y > x. Then, $\frac{x+y}{2}$ is a rational number lying between x and y.

Ex.2 : Find a rational number lying between 3 and 4.

Sol. Here, 4 > 3

We know that, if x and y are two numbers such that y > x.

Then, $\frac{x+y}{2}$ is a rational number between x and y.

So, a rational number between 3 and 4 = $\frac{3+4}{2}=\frac{7}{2}$

- **When more than one rational numbers are determined**

Let x and y be two rational numbers, such that y > x and we want to find n rational numbers between x and y.

Then, n rational numbers lying between x and y are (x + d), (x + 2d), (x + 3d) ,......, (x + nd), where d = $\frac{y-x}{n+1}$.

Knowledge Plus: Rational number/numbers between any two rational numbers is/are written in the standard form

Ex.3 : Find four rational numbers lying between $\frac{3}{5}$ and $\frac{2}{3}$.

Sol. Here, $\frac{2}{3} > \frac{3}{5}$

So, the four rational numbers between them will be (x + d), (x + 2d), (x + 3d) and (x + 4d)

where, $d = \frac{y-x}{n+1}$ and n = numbers of rational numbers to find between x and y.

Find the value of d $\frac{y-x}{n+1}$, where d =. Now $d = \frac{\frac{2}{3}-\frac{3}{5}}{4+1} = \frac{\frac{10-9}{15}}{5} = \frac{1}{15\times 5} = \frac{1}{75}$,

Here, $x + d = \frac{3}{5}+\frac{1}{75} = \frac{45+1}{75} = \frac{46}{75}$; $x + 2d = \frac{3}{5}+\frac{2}{75} = \frac{45+2}{75} = \frac{47}{75}$

$x + 3d = \frac{3}{5}+\frac{3}{75} = \frac{45+3}{75} = \frac{48}{75} = \frac{16}{25}$ and $x + 4d = \frac{3}{5}+\frac{4}{75} = \frac{45+4}{75} = \frac{49}{75}$

Hence, the required four rational numbers lying between $\frac{3}{5}$ and $\frac{2}{3}$ are $\frac{46}{75}, \frac{47}{75}, \frac{16}{25}$ and $\frac{49}{75}$ and.

Ex.4: Find four rational numbers between –5 and – 6.

Sol. $d = \frac{y-x}{n+1} = \frac{-5-(-6)}{4+1} = \frac{-5+6}{5} = \frac{1}{5}$

i.e., $\left(-6+\frac{1}{5}\right), \left(-6+\frac{2}{5}\right), \left(-6+\frac{3}{5}\right)$ and $\left(-6+\frac{4}{5}\right)$

$= \left(\frac{-30+1}{5}\right), \left(\frac{-30+2}{5}\right), \left(\frac{-30+3}{5}\right)$ and $\left(\frac{-30+4}{5}\right)$ and $= -\frac{29}{5}, -\frac{28}{5}, -\frac{27}{5}$ and $-\frac{26}{5}$

1.1.2 Decimal expansion of rational numbers

Every rational number can be expressed as terminating decimal or non-terminating but repeating decimals.

Terminating decimal (The remainder becomes zero)

The word "terminate" means "end". A decimal that ends is a terminating decimal.

OR

A terminating decimal doesn't keep going. A terminating decimal will have finite number of digits after the decimal point.

$\frac{3}{4} = 0.75, \frac{8}{10} = 0.8, \frac{5}{4} = 1.25, \frac{25}{16} = 1.5625$

Ex. 5: Express7/8 in the decimal form.

Sol. We have, $\therefore$

```
8)7.000(0.875
  64
  60
  56
   40
   40
    0
```

$\frac{7}{8} = 0.875$

Non-terminating & repeating (recurring decimal)

(The remainder never becomes zero)

A decimal in which a digit or a set of finite number of digits repeats periodically is called Non-terminating repeating (recurring) decimals.

$\frac{5}{3} = 1.6666........ = 1.\overline{6}$, $\frac{7}{11} = 0.636363....... = 0.\overline{63}$,

$\frac{1}{999} = 0.001001001...... = 0.\overline{001}$

Ex. 6 : Express 2/11 as a decimal fraction.

Sol. We have,

$$
\begin{array}{r|l}
11 & 2.00 \\
 & \underline{11} \\
 & \;\;90 \\
 & \;\;\underline{88} \\
 & \;\;\;20 \\
 & \;\;\;\underline{11} \\
 & \;\;\;\;90 \\
 & \;\;\;\;\underline{88} \\
 & \;\;\;\;\;20 \\
 & \;\;\;\;\;\underline{11} \\
 & \;\;\;\;\;\;90 \\
 & \;\;\;\;\;\;\underline{88} \\
 & \;\;\;\;\;\;\;2
\end{array}
$$

$\therefore \dfrac{2}{11} = 0.181818..... = 0.\overline{18}$

Ex. 7: If, $\dfrac{1}{7} = 0.\overline{142857}$ write the decimal expansion of $\dfrac{2}{7}, \dfrac{3}{7}, \dfrac{4}{7}$, and $\dfrac{5}{7}$ without actually doing the long division.

Sol. Thus, we have $\dfrac{2}{7} = 2 \times \dfrac{1}{7} = 0.\overline{285714}$; $\dfrac{3}{7} = 3 \times \dfrac{1}{7} = 0.\overline{425871}$;

$\dfrac{4}{7} = 4 \times \dfrac{1}{7} = 0.\overline{571428}$; $\dfrac{5}{7} = 5 \times \dfrac{1}{7} = 0.\overline{714285}$

- **Method to convert non-terminating decimal to the form p/q.**

  In a non-terminating decimal, we have two types of decimal representations

  (a) Pure recurring decimal

  (b) Mixed recurring decimal

(a) Pure recurring decimal

It is a decimal representation in which all the digits after the decimal point are repeated.

Following are the steps to convert it in the form p/q.

- Denote pure recurring decimal as x.
- Write the number in decimal form by removing bar from top of repeating digits.
- Count the number of digits having bar on their heads.
- Multiply the repeating decimal by 10, 100, 1000, ... depending upon 1 place repetition, 2 place repetition, 3 place repetition and so on present in decimal number.
- Subtract the number obtained in step 2 from a number obtained in step 4.
- Find the value of x in the form p/q.

(b) Mixed recurring decimal

It is a decimal representation in which there are one or more digits present before the repeating digits after decimal point. Following are the steps to convert it to the form p/q.

- Denote mixed recurring decimal as x.
- Count the number of digits after the decimal point which do not have bar on them. let it be 'n'.
- Multiply both sides of x by 10^n to get only repeating decimal numbers on the right side of the decimal point.
- Further use the method of converting pure recurring decimal to the form p/q and get the value of x.

Ex. 8 : Express each of the following pure recurring decimals in the form p/q.

(i) $0.\overline{6}$ (ii) $0.\overline{585}$ (iii) $23.\overline{43}$

Sol. (i) Let x = $0.\overline{6}$ $\Rightarrow$ x = 0.666 .... (i)

Here, we have only one repeating digit, so we multiply both sides of (i) by 10 to get

$\Rightarrow$ 10x = 6.66 ....(ii)

On subtracting (i) from (ii), we get : 10x – x = (6.66......) – (0.66 ....)

$\Rightarrow 9x = 6 \Rightarrow x = \frac{6}{9} \Rightarrow x = \frac{2}{3}$, Hence $0.\overline{6} = \frac{2}{3}$,

(ii) Let x = $0.\overline{585} \Rightarrow x = 0.585585585$.......... $0.\overline{585} = \frac{65}{111}$ ...(i)

Here, we have three repeating digits, wo we multiply both sides of (i) by 10^3 = 1000 to get

$\Rightarrow$ 1000x = 585.585585 ...(ii)

On subtracting (i) from (ii), we get

1000x – x = (585.585585..........) – (0.585585......)

$\Rightarrow$ 999x = 585

$\Rightarrow$ x = $\Rightarrow$ 999x = 585, Hence, $0.\overline{585} = \frac{585}{999}$

(iii) Let x = $23.\overline{43} \Rightarrow$ x = 23.434343 ...(i)

Multiplying both sides of (i) by 100, we get 100x = 2343.4343......

$\Rightarrow$ 99x = 2320 $\Rightarrow$ x = $\frac{2320}{99}$

Hence, $23.\overline{43} = \frac{2320}{99}$

Alter Method

We have, $23.\overline{43} = 23 + 0.\overline{43} = 23 + \frac{43}{99}$ [Using the above rule, we have $0.\overline{43} = \frac{43}{99}$]

$\Rightarrow 23.\overline{43} = \frac{23 \times 99 + 43}{99} = \frac{2277 + 43}{99} = \frac{2320}{99}$

Ex. 9: Express the following mixed recurring decimals in the form p/q.

(i) $0.3\overline{2}$ (ii) $0.12\overline{3}$ (iii) $15.7\overline{12}$

Sol. (i) Let x = $0.3\overline{2}$

Clearly, there is just one digit on the right side of the decimal point which is without bar. So, we multiply both sides of x by 10 so that only the repeating decimal is left on the right side of the decimal point.

$\therefore$ 10x = $3.\overline{2}$ $\Rightarrow$ 10 x = 3 + $\frac{2}{9}$ $\Rightarrow \frac{9 \times 3 + 2}{9} \Rightarrow 10x = \frac{29}{9}$ 10 x $\Rightarrow$ x = $\frac{29}{90}$

(ii) Let x = $0.12\overline{3}$

Clearly, there are two digits on the right side of the decimal point which is without bar. So, we multiply both sides of equation by $10^2 = 100$ so that only the repeating decimal is left on the right side of the decimal point.

$$\therefore\ 100x = 12.\overline{3} \Rightarrow 100x = 12 + 0.\overline{3} \Rightarrow 100x = 12 + \frac{3}{9}$$

$$\Rightarrow 100x = \frac{12 \times 9 + 3}{9} \Rightarrow 100x = \frac{108 + 3}{9} \Rightarrow 100x = \frac{111}{9} \Rightarrow x = \frac{111}{900} = \frac{37}{300}$$

(iii) Let x = $15.7\overline{12}$; $10x = 157.\overline{12}$

$$\Rightarrow 10x = 157 + 0.\overline{12} \Rightarrow 10x = 157 + \frac{12}{99} \Rightarrow 10x = 157 + \frac{4}{33}$$

$$\Rightarrow 10x = \frac{157 \times 33 + 4}{33} \Rightarrow 10x = \frac{5181 + 4}{33} \Rightarrow \frac{5185}{33}\ 10x \Rightarrow x = \frac{5185}{330} = \frac{1037}{66}$$

1.2 IRRATIONAL NUMBERS

The numbers, which cannot be expressed in the form p/q, where p, q both are integers and q $\neq$ 0, are called irrational numbers. **Ex.** $\sqrt{2}, \sqrt{3}, \pi$ etc.

> **Knowledge Plus:**
> - When we use the symbol '$\sqrt{\ }$', we assume that it is the positive square root of the number. So $\sqrt{4} = 2$ though both 2 and –2 are square roots of 4.
> - There are infinitely many irrational numbers between any two given irrational numbers.

1.2.1 Decimal Expansions of Irrational Numbers

The decimal expansion of an irrational number is non-terminating non-recurring. In order words, a number whose decimal expansion is non-terminating non-recurring is irrational.

Ex. Let $\sqrt{2}$ and π be two irrational numbers, then their decimal expansions are

= 1.4142135623730950488016887242096... and π = 3.14159265358979323846264338327950...

We often take value of π as $\frac{22}{7}$, which is a rational number but $\pi \neq \frac{22}{7}$.

Ex. 10: Find an irrational number between $\frac{1}{7}$ and $\frac{1}{3}$.

Sol. To find irrational number, firstly we will divide 1 by 7 and 1 by 3.

Now,

```
7) 10 (0.142857...
    7
    30
    28
     20
     14
      60
      56
       40
       35
        50
        49
         1
```

$\therefore\ \frac{1}{7} = 0.142857... = 0.\overline{142857}$

Now,

```
3) 10 (0.33...
    9
    10
     9
     1
```

Thus, $\frac{1}{3} = 0.333... = 0.\overline{3}$

That means the required irrational numbers will lie between $0.\overline{142857}$ and $0.\overline{3}$. Also, the irrational numbers have non-terminating non-repeating decimals. Hence, the required irrational number between $\frac{1}{7}$ and $\frac{1}{3}$ is 0.2101001000.....

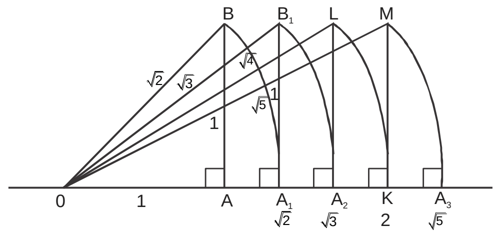
1.2.2 Representation of irrational number on the number line

Consider the number line and mark a point O on it let it represent zero. Let A represent 1 on the number line. So, OA = 1. At A draw AB perpendicular to OA.

Let AB=OA = 1

$\therefore$ By Phythagoras Theorem,

$$OB = \sqrt{(OA)^2 + (AB)^2} = \sqrt{(1)^2 + (1)^2} = \sqrt{1+1} = \sqrt{2}$$



Taking O as centre and radius = OB = $\sqrt{2}$, draw a circle cutting the number line at A_1, where

OA_1 = OB = $\sqrt{2}$

$\Rightarrow$ A_1 represents on number line. Now draw A_1B_1 perpendicular to number line at A_1 and let A_1B_1 = 1

$\therefore$ $OB_1 = \sqrt{(OA_1)^2 + (A_1B_1)^2} = \sqrt{(\sqrt{2})^2 + (1)^2} = \sqrt{2+1} = \sqrt{3}$

Taking O as centre and OB_1= as radius, draw a circle cutting the number line at A_2 where $OA_2 = OB_1 = \sqrt{3}$

$\Rightarrow$ A_2 represents $\sqrt{3}$ on number line Continue this process and get the point K on number line where

$$OK = OL = \sqrt{(OA_2)^2 + (A_2L)^2} = \sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{3+1} = 2$$

$\Rightarrow$ K represents 2 on number line.

Again, get a point A_3 on number line where

$$OA_3 = OM = \sqrt{(OK)^2 + (KM)^2} = \sqrt{(\sqrt{2})^2 + (1)^2} = \sqrt{4+1} = \sqrt{5}$$

$\Rightarrow$ A_3 represents $\sqrt{5}$ on number line.

In this way, we can show that there exists points on Number Line representing $\sqrt{5}, \sqrt{7}, \sqrt{8}$ etc. which are irrational numbers.

In fact for every irrational, there exists a unique point on the number line.

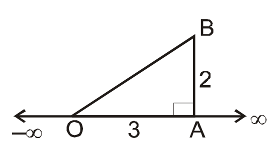
EX.11 : Locate $\sqrt{13}$ on the number line.

Sol. Write the given number (without root) as the sum of the squares of two natural numbers (say a and b, where a $\geq$ b).

Here, 13 = 9 + 4 = $3^2 + 2^2$ So, we take a = 3 and b = 2

- Take the length equal to these two natural numbers on the number line ('a' on number line and 'b' vertically) such that one is perpendicular to other.

  Draw OA = 3 units on number line and then draw AB = 2 units, such that AB $\perp$ OA. Join OB.



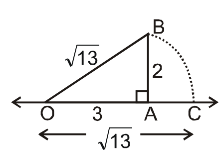
- By pythagoras theorem, find OB.

  OB = $\sqrt{OA^2 + AB^2} = \sqrt{3^2 + 2^2} = \sqrt{9+4} = \sqrt{13}$

- Taking O as centre and OB as radius, draw an arc to represent required irrational number.

  Taking O as centre and radius equal to OB, draw an arc, which cuts the number line at C.

  Hence, OC represents $\sqrt{13}$.



1.2.3 Representation of $\sqrt{x}$ for any given positive real number x on number line

(i) Let x be a positive real number. Take AB = x units and BC = 1 unit on the real line ℓ.

(ii) Find the mid point O of AC and draw a semicircle with centre O and radius OA or OC.

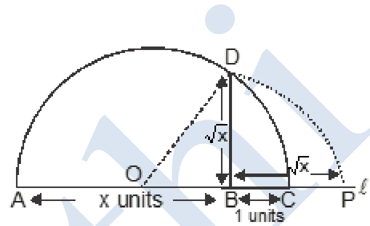
(iii) At B, draw a line BD$\perp$ AC, where D is a point on the semicircle.

(iv) Join OD.

(v) Further, with centre B and radius BD, draw an arc intersecting the real line ℓ at P.

Therefore, BP = BD = $\sqrt{x}$.

Justification: We have, In right triangle OBD,



OD = OA = OC = $\dfrac{x+1}{2}$ units $\left(\dfrac{x+1}{2}\right)$ (radius of the semicircle)

OB = OC –BC = $\left(\dfrac{x+1}{2}-1\right)$ units = $\left(\dfrac{x+1}{2}\right)$ units

In right ΔOBD, we have $OD^2 = OB^2 + BD^2$

$BD^2 = OD^2 - OB^2$

and BD = $\sqrt{OD^2 - OB^2}$ [By Pythagoras theorem]

$= \sqrt{\left(\dfrac{x+1}{2}\right)^2 - \left(\dfrac{x-1}{2}\right)^2}$ units $= \sqrt{\left(\dfrac{x+1}{2}+\dfrac{x-1}{2}\right)\left(\dfrac{x+1}{2}+\dfrac{x-1}{2}\right)}$ units [$\therefore$ $A^2 - B^2 = (A+B)(A-B)$]

$= \sqrt{x \times 1}$ units = $\sqrt{x}$ units. So, BD = $\sqrt{x}$ units.

Thus, $\sqrt{x}$ exists for all positive real numbers.

Hence, the point P represents $\sqrt{x}$ on the real line.

1.3 SQUARE & SQUARE ROOTS

The second power of number is called the square of that number. In other words, the square of a number is the product of the number with the number itself.

A given number is a perfect square, if it is expressed as a product of pairs of equal factors.

Important properties

(i) A natural number having 2, 3, 7 or 8 in the unit's place is never a perfect square (or squared number).

Ex. 17, 23, 118, 222 are not perfect squares.

(ii) The square of an even number is always an even number.

Ex. $2^2 = 4$, $6^2 = 36$, $10^2 = 100$, $12^2 = 144$.

(iii) The square of an odd number is always an odd number.

Ex. $3^2 = 9$, $7^2 = 49$, $13^2 = 169$, $15^2 = 225$.

(iv) The number of zeroes at the end of a perfect square is never odd.

Ex. 100, 400, 3600, 640000 are perfect squares and 1000, 4000, 6400000 are not perfect squares.

(v) The square of a natural number n is equal to the sum of the first n odd numbers.

Ex. $1^2 = 1$ = sum of the first 1 odd number.

$2^2 = 1 + 3$ = sum of the first 2 odd numbers.

$3^2 = 1 + 3 + 5$ = sum of the first 3 odd numbers.

(vi) For every natural number n, difference of square of any two consecutive natural numbers is prime.

Ex. $(n + 1)^2 - n^2 = (n + 1 + n)(n + 1 - n) = (n + 1) + n$

$4^2 - 3^2 = (3 + 1) + 3 = 7$.

$16^2 - 15^2 = (15 + 1) + 15 = 31$.

(vii) A perfect square (other than 1) is either a multiple of 3 or exceeds a multiple of 3 by 1.

Ex. $49 = (7)^2 = 3 \times 16 + 1$, $169 = (13)^2 = 3 \times 56 + 1$.

(viii) A perfect square (other than 1) is either a multiple of 4 or exceeds a multiple of 4 by 1.

Ex. $441 = (21)^2 = 4 \times 110 + 1$.

➢ **Some other properties**

(i) If the unit digit of the number is zero then the unit digit of the square of this number will also be zero and the number of zeros will be double in the square than that of its root.

Ex. $(60)^2 = 3600$, $(130)^2 = 16900$

(ii) If the unit digit of the number is 5 then the unit digit of its square is also 5 and the number formed by last two digits is 25.

Ex. $(35)^2 = 1225$, $(45)^2 = 2025$, $(55)^2 = 3025$ etc.

(iii) If the unit digit of any number is 1 or 9 then the unit digit of the square of its number is always 1.

Ex. $(71)^2 = 5041$, $(31)^2 = 961$, $(19)^2 = 361$

(iv) If the unit digit of any number is 2 or 8 then the unit digit of the square of its number is always 4.

(v) If the unit digit of any number is 2 or 7 then the unit digit of its square is always 9.

Ex. $(23)^2 = 529$, $(27)^2 = 729$.

(vi) If the unit digit of any number is 4 or 6 then the unit digit of its square is always 6.

Ex. $(26)^2 = 676$, $(24)^2 = 576$

$(14)^2 = 196$, $(16)^2 = 256$ etc.

(vii) The square of any number is always positive irrespective of the nature of the given number.

(viii) Non square numbers between the squares of two consecutive natural numbers n and

$n + 1 \rightarrow (n + 1)^2 - n^2 - 1 = n^2 + 1 + 2n - n^2 - 1 = 2n$

(ix) If a natural number cannot be expressed as a sum of successive odd natural numbers starting with 1, then it is not a perfect square.

(x) Square root of a negative integer is an imaginary number.

(xi) Square roots of integers that are not perfect squares are always irrational numbers.

(xii) Every composite number can be uniquely factored as a product of prime numbers only.

(xiii) If a perfect square is of n-digits, then its square root will have n/2 digits if n is even or $\frac{(n+1)}{2}$ if n is odd.

1.4 CUBE & CUBE ROOTS

Cube of a number is obtained by multiplying the number itself thrice.

Ex. 27 is the cube of 3 as $27 = 3 \times 3 \times 3$.

Cube root

Cube root of a given number is that number which when raised to the third power produces the given numbers, that is the cube root of a number x is the number whose cube is x^3.

The cube root of x is written as $\sqrt[3]{x}$.

Ex. cube root of 64 is 4 as $4 \times 4 \times 4 = 64$

• **Short-cut method of finding cube roots of exact cubes consisting of up to 6 digits :**

Before we discuss the method to find the cube roots of exact cubes, the following two remarks are very useful and must be remembered by heart.

Clearly from above

$1\leftrightarrow1$, $4\leftrightarrow4$, $5\leftrightarrow5$, $6\leftrightarrow6$, $9\leftrightarrow9$, $0\leftrightarrow0$, $2\leftrightarrow8$, $3\leftrightarrow7$.

➢ **Some other properties**

(i) Cubes of all odd natural numbers are odd.

(ii) Cubes of all even natural numbers are even.

(iii) The cube of a natural number which is a multiple of 3 is a multiple of 27.

(iv) The cube of a natural number which is of the form 3n + 1 (e.g., 4, 7, 10......) is also a number of the form 3n + 1.

(v) The cube of a natural number which is of the form 3n + 2 (e.g., 5, 8, 11......) is also a number of the form 3n + 2.

1.5 TEST OF DIVISIBILITY

- **Divisibility by 2 :** A number is divisible by 2 if the unit's digit is zero or divisible by 2.

Ex. 4, 12, 30, 18, 102, etc. are all divisible by 2.

- **Divisibility by 3 :** A number is divisible by 3 if the sum of digits in the number is divisible by 3.

Ex. The number 3792 is divisible by 3 since 3 + 7 + 9 + 2 = 21, which is divisible by 3.

- **Divisibility by 4 :** A number is divisible by 4 if the number formed by the last two digits (ten's digit and unit's digit) is divisible by 4 or are both zero.

Ex. The number 2616 is divisible by 4 since 16 is divisible by 4.

- **Divisibility by 5 :** A number is divisible by 5 if the unit's digit in the number is 0 or 5.

Ex. 13520, 7805, 640, 745, ect. are all divisble by 5.

- **Divisibility by 6 :** A number is divisible by 6 if the number is even and sum of its digits is divisible by 3.

Ex. The number 4518 is divisible by 6 since it is even and sum of its digits 4+5+1+8=18 is divisible by 3.

- **Divisibility by 7 :** The unit digit of the given number is doubled and then it is subtracted from the number obtained after omitting the unit digit. If the remainder is divisible by 7, then the given number is also divisible by 7.

Ex. consider the number 448. On doubling the unit digit 8 of 448 we get 16.
Then, 44–16 = 28.
Since 28 is divisible by 7, 448 is divisible by 7.

- **Divisibility by 8 :** A number is divisible by 8, if the number formed by the last 3 digits is divisible by 8.

Ex. The number 41784 is divisible by 8 as the number formed by last three digits i.e. 784 is divisible by 8.

- **Divisibility by 9 :** A number is divisible by 9, if the sum of its digits is divisible by 9.

Ex. The number 19044 is divisible by 9 as the sum of its digits 1 + 9 + 0 + 4 + 4 = 18 is divisible by 9.

- **Divisibility by 10 :** A number is divisible by 10, if ends in zero.

Ex. The last digit of 580 is zero, therefore 580 is divisible by 10.

- **Divisibility by 11 :** A number is divisible by 11, if the difference of the sum of the digits at odd places and sum of the digits at even places is either zero or divisible by 11.

Ex. In the number 38797, the sum of the digits at odd places is 3 + 7 + 7 = 17 and the sum of the digits at even places is 8 + 9 = 17. The difference is 17 – 17 = 0, so the number is divisible by 11.

- **Divisibility by 12 :** A number is divisible by 12, if it is divisible by 3 and 4.

1.6 H.C.F. & L.C.M. OF NUMBERS

- **The least number which when divided by d_1, d_2 and d_3 leaves the remainders r_1, r_2 and r_3 respectively, such that $(d_1 - r_1) = (d_2 - r_2) = (d_3 - r_3)$ is (L.C.M. of d_1, d_2 and d_3) – $(d_1 - r_1)$ or $(d_2 - r_2)$ or $(d_3 - r_3)$.**

EX.12 Find the least number which when divided by 9, 10 and 15 leaves the remainders 4, 5 and 10, respectively.

Sol. Here, 9 – 4 = 10 – 5 = 15 – 10 = 5
Also, L.C.M. (9, 10, 15) = 90
∴ the required least number = 90 – 5 = 85.

- **A number on being divided by d_1 and d_2 successively leaves the remainders r_1 and r_2, respectively. If the number is divided by $d_1 \times d_2$, then the remainder is $(d_1 \times r_2 + r_1)$.**

EX.13 : A number on being divided by 10 and 11 successively leaves the remainders 5 and 7, respectively. Find the remainder when the same number is divided by 110.

Sol. The required remainder = $d_1 \times r_2 + r_1 = 10 \times 7 + 5 = 75$.

- **To find the number of numbers divisible by a certain integer.**

EX.14 : (i) How many numbers up to 532 are divisible by 15?

(ii) How many numbers up to 300 are divisible by 5 and 7 together?

Sol. (i) We divide 532 by 15.

$532 = \underline{35} \times 15 + 7$

The quotient obtained is the required number of numbers. Thus there are 35 such numbers.

(ii) L.C.M. of 5 and 7 = 35

We divide 300 by 35

$300 = \underline{8} \times 35 + 20$

Thus there are 8 such numbers.

- **Two numbers when divided by a certain divisor give remainders r_1 and r_2. When their sum is divided by the same divisor, the remainder is r_3. The divisor is given by $r_1 + r_2 - r_3$.**

EX.15 : Two numbers when divided by a certain divisor give remainders 473 and 298, respectively. When their sum is divided by the same divisor, the remainder is 236. Find the divisor.

Sol. The required divisor = 437 + 298 – 236 = 499.

- **Product of two numbers = L.C.M. of the numbers × H.C.F. of the numbers.**

EX.16 : The H.C.F. and the L.C.M. of any two numbers are 63 and 1260, respectively. If one of the two numbers is 315, find the other number.

Sol. The required number $= \dfrac{\text{L.C.M.} \times \text{H.C.F.}}{\text{First number}} = \dfrac{1260 \times 63}{315} = 252$

- **To find the greatest number that will exactly divide x, y and z. Required number = H.C.F. of x, y, and z.**

EX.17 : Find the greatest number that will exactly divide 200 and 320.

Sol. The required greatest number = H.C.F. of 200 and 320 = 40.

- **To find the greatest number that will divide x, y, and z leaving remainders a, b and c, respectively. Required number = H.C.F. of (x – a), (y – b) and (z – c).**

EX.18 : Find the greatest number that will divide 148, 246 and 623 leaving remainders 4, 6 and 11, respectively.

Sol. The required greatest number = H.C.F. of (148 – 4), (246 – 6) and (623 – 11),

i.e. H.C.F. of 144, 240 and 612 = 12.

- **To find the least number which is exactly divisible by x, y and z. Required number = L.C.M. of x, y and z.**

EX.19 : What is the smallest number which is exactly divisible by 36, 45, 63 and 80?

Sol. The required smallest number = L.C.M. of 36, 45, 63 and 80 = 5040.

- **To find the least number which when divided by x, y and z leaves the remainder a, b and c, respectively. It is always observed that (x – a) = (y – b) = (z – c) = k(say)**

$\therefore$ **Required number = (L.C.M. of x, y and z)–k.**

EX.20 : Find the least number which when divided by 36, 48 and 64 leaves the remainders 25, 37 and 53, respectively.

Sol. Since (36–25) = (48–37) = (64–53) = 11, therefore the required smallest number

= (L.C.M. of 36, 48 and 64) – 11

= 576 – 11 = 565

- **To find the least number which when divided by x, y and z leaves the same remainder r in each case, Required number = (L.C.M. of x, y and z)+r.**

EX.21 : Find the least number which when divided by 12, 16 and 18, will leave in each case a remainder 5.

Sol. The required smallest number

= (L.C.M. of 12, 16 and 18) + 15

= 144 + 5 = 149.

- **To find the greatest number that will divide x, y and z leaving the same remainder in each case.**

  (a) When the value of remainder r is given : Required number = H.C.F. of (x – r), (y – r) and (z – r).

  (b) When the value of remainder is not given : Required number = H.C.F. of |(x – y)|, |(y – z)| and |(z – x)|

EX.22 : (a) Find the greatest number which will divide 772 and 2778 so as to leave the remainder 5 in each case.

(b) Find the greatest number which on dividing 152, 277 and 427 leaves remainder.

Sol. (a) The required greatest number

= H.C.F. of (772 – 5) and (2778 – 5)

= H.C.F. of 767 and 2773 = 59.

(b) The required greatest number.

= H.C.F. of |(x – y)|, |(y – z)| and |(z – x)|

= H.C.F. of |(152 – 27)|, |(277 – 427)| and |(427 – 152)|

= H.C.F. of 125, 275 and 150 = 25.

- **To find the n-digit greatest number which, when divided by x, y and z.**

  (a) leaves no remainder (i.e., exactly divisible)

  $\Rightarrow$ **L.C.M. of x, y and z = L**

  $$\Rightarrow \frac{L\overline{)\text{n} - \text{digit greatest number}}}{\text{Remainder} = \text{R}}$$

  $\Rightarrow$ **Required number = n-digit greatest number – R**

  (b) leaves remainder K in each case. Required number = (n-digit greatest number-R) + K.

EX.23 : Find the greatest number of 4 digits which, when divided by 12, 18, 21 and 28, leaves 3 as a remainder in each case.

Sol. L.C.M. of 12, 18, 21 and 28 = 252.

$$\begin{array}{r|l} 252\,) & 9999\,(39 \\ & \underline{9828} \\ & \;\;171 \end{array}$$

$\therefore$ The required number = (9999 – 171) + 3 = 9931.

- **To find the n-digit smallest number which when divided by x, y and z.**

  (a) leaves no remainder (i.e., exactly divisible)

  $\Rightarrow$ **L.C.M. of x, y and z = L**

  $$\Rightarrow \frac{L\overline{)\text{n} - \text{digit smallest number}}}{\text{Remainder} = \text{R}}$$

  $\Rightarrow$ **Required number = n-digit smallest number + (L–R)**

  (b) leaves remainder K in each case. Required number = n-digit smallest number + (L-R) + k.

EX.24 : (a) Find the least number of four digits which is divisible by 4, 6, 8 and 10.

(b) Find the smallest 4-digit number, such that when divided by 12, 18, 21 and 28, it leaves remainder 3 in each case.

Sol. **(a)** L.C.M. of 4, 6, 8 and 10 = 120.

$$\begin{array}{r|l} 120\,) & 1000\,(8 \\ & \underline{\;960} \\ & \;\;\;40 \end{array}$$

$\therefore$ The required number = 1000 + (120 – 40) = 1080.

(b) L.C.M. of 12, 18, 21 and 28 = 252.

$\therefore$ The required number = 1000 + (252 – 244) + 3 = 1011.

1.7 NUMBER OF FACTORS OF A GIVEN NUMBER

Factors of a number N refers to all the numbers which divide N completely. These are also called divisors of a number.

There are certain basic formulaes pertaining to factors of a number N such that,

$N = p^a q^b r^c$; where p, q, r are prime factors of the number 'n' and a, b, c are non-negative powers / exponents.

Number of factors of N = (a + 1) (b + 1) (c + 1)

Product of factors of **N = N**$^{(\text{No. of factors} / 2)}$

Sum of factors = $\dfrac{(p^0+p^1+....+p^a)(q^0+q^1+....+q^b)(r^0+r^1+....+r^c)}{(p-1)(q-1)(r-1)}$

EX.25 : Consider the number 120. Find the following for N.
(i) Sum of factors (ii) Number of factors (iii) Product of factors

Sol. The prime factorisation of 120 is $2^3 \times 3^1 \times 5^1$

By applying formula :

(i) Sum of factors = $= \dfrac{\left[(2^0+2^1+2^2+2^3)(3^0+3^1)(5^0+5^1)\right]}{(2-1)(3-1)(5-1)} = 45$

(ii) Number of factors = (3 + 1) × (1 + 1) × (1 + 1) = 4 × 2 × 2 = 16

(iii) Product of factors = $(120)^{16/2} = 120^8$

EX.26 : Find the following for the number 84
(i) Number of odd factors (ii) Number of even factors

Sol. $84 = 2^2 \times 3^1 \times 7^1$

Total number of factors = $(2+1)(1+1)(1+1) = 12$

(i) Number of odd factors will be all possible combinations of power of 3 and 5 (excluding any power of 2).
Hence number of odd factors = (1 + 1) (1 + 1) = 4

(ii) Number of even factors = Total no. of factors – no. of odd factors = 12 – 4 = 8

1.7.1 Number of ways of expressing a composite number as a product of two factors

Let us consider an EX. of small composite number say, 24

then 24 = 1 × 24
2 × 12
3 × 8
4 × 6

So it is clear that

The number of ways of expressing a composite no. as a product of two factors = $\dfrac{1}{2}$ **× the no. of total factors.**

EX.27 : Find the number of ways of expressing 180 as a product of two factors.

Sol. $180 = 2^2 \times 3^3 \times 5^1$

Number of factors = (2 + 1) (2 + 1) (1 + 1) = 18

Hence, there are total $\dfrac{18}{2} = 9$ ways in which 180 can be expressed as a product of two factors.

Knowledge Plus: As you know when you express any perfect square number 'N' as a product of two factors namely $\sqrt{N}$ and $\sqrt{N}$,and also know that since in this case $\sqrt{N}$ appears two times but it is considered only once while calculating the no. of factors so we get always an odd number as number of factors so we can not divide the odd number exactly by 2 as in the above formula. So if we have to consider these two same factors then we find the number of ways of expressing N as a **product of two factors=**$\dfrac{\text{Number of factors} + 1}{2}$

Again if it is asked that find no. of ways of expressing N as a product of two distinct factors then we do not consider 1 way (i.e. N = $\sqrt{N} \times \sqrt{N}$) then **no. of ways**) = $\dfrac{\text{number of factors } - 1}{2}$

EX.28 : (a) Find the number of ways expressing 36 as a product of two factors.
(b) In how many ways can 576 be expressed as the product of two distinct factors?

Sol. (a) $36 = 2^2 \times 3^2$

Number of factors = (2 + 1) (2 + 1) = 9

Hence, the no. of ways expressing 36 as a product of two factors $= \frac{(9+1)}{2} = 5$

as $36 = 1 \times 36,\ 2 \times 18, 3 \times 12, 4 \times 9$ and 6×6

(b) $576 = 2^6 \times 3^2$

$\therefore$ Total number of factors $= (6+1)(2+1) = 21$

So the number of ways of expressing 576 as a product of two distinct factors $= \frac{21-1}{2} = 10$

Knowledge Plus: Since the word 'distinct' has been used therefore we do not include $576 = 26 \times 26$.

1.8 CYCLICITY

The concept of cyclicity is used to identify the last digit of the number. This concept utilize the fact that remainders repeat themselves after a certain interval when divided by a number.

First of all, we know that remainder = 0 to d – 1,

where d = number by which the divisor is divided. If we divide a^n by d, the remainder can be any value from 0 to d–

1. If we keep on increasing the value of n, the remainders are cyclical in nature. The pattern of the remainders would repeat.

Number	Cyclicity
2, 3, 7, 8	4
5, 6	1
4, 9	2

EX.29 : Find the units place digit of 2^{99}.

Sol. Divide the exponent by 4 i.e.; $\frac{99}{4}$ gives remainder 3. That means; unit place digit of $2^{99} = 2^3 = 8$

EX.30 : Find the units place digit of $2^{43^{44}}$.

Sol. There cyclicity of units place digit is 4 (units place digit is 2, from the above table we can see the cyclicity of 2 is 4). Now we have to find the remainder when exponent of 2 is divided by 4, that is the remainder when 43^{44} is divided by 4. Remainder of $\frac{43^{44}}{4} = \frac{(44-1)^{44}}{4}$

So, term $\frac{1^{44}}{4}$ is remaining. Remainder of $\frac{1^{44}}{4}$ = 1

Remainder = 1, units place = 2^1 = 2.

SUBJECTIVE QUESTIONS

Q.1 Simplify $\sqrt{2}+4\sqrt{2}+6\sqrt{2}$

Q.2 Simplify $\sqrt{75}-2\sqrt{27}+5\sqrt{12}$

Q.3 Find the product of $5\sqrt{2}$ and $6\sqrt{3}$

Q.4 Divide $\sqrt{108}$ by $\sqrt{147}$

Q.5 What kind of decimal expansions do the following numbers have?

$\frac{5}{8};\frac{22}{3};\frac{1}{7}.$

Q.6 Insert 5 rational numbers between 3 and 4.

Q.7 Insert three irrational numbers between 4 and 5.

Q.8 Represent the number $\frac{3}{5}$ on the number line.

Q.9 Find a fraction between $\frac{3}{8}$ and $\frac{2}{5}$.

Q.10 Which of the following fractions yield a recurring decimal?

$\frac{5}{3};\frac{7}{16};\frac{9}{14};\frac{5}{7};\frac{12}{5};\frac{6}{11}$

Q.11 Find an irational number between $\frac{1}{5}$ and $\frac{5}{16}$.

Q.12 Which of the following numbers are not rational?

1.256; 0.45454545.....,

0.0500500050005.....;

5.51551555151....;

2.012340123401234...

Q.13 Represent 1.129129129 .... as a fraction.

Q.14 What is the simplest form of $\sqrt{200}-\sqrt{50}$?

Q.15 Rationalise the denominator of $\frac{5}{\sqrt{10}+\sqrt{5}}$.

Q.16 If x = $\sqrt{2}-1$ what is the value of x – 1/x?

Q.17 Simplify $(\sqrt{5}+1)^2+(\sqrt{5}-1)^2$.

Q.18 Simplify the following :

(i) $(3+\sqrt{2})(2+\sqrt{3})$ (ii) $(5+\sqrt{3})(5-\sqrt{3})$

(iii) $(7+2\sqrt{2})^2$ (iv) $(\sqrt{5}-\sqrt{3})^2$

Q.19 Find the value of the following :

(i) $81^{1/4}$ (ii) $64^{1/3}$

(iii) $32^{3/5}$ (iv) $27^{2/3}$

Q.20 Simplify the following :

(i) $3^{1/5}\,.\,3^{2/5}$ (ii) $\left(\frac{4}{9}\right)^{1/2}$

(iii) $2^{2/3}.3^{2/3}$ (iv) $(2^{1/3})^3$

Q.21 Represent $\sqrt{6}$ on the number line.

Q.22 Find two irrational numbers between $\sqrt{5}$ and $\sqrt{6}$.

Q.23 If x = $\frac{\sqrt{5}-2}{\sqrt{5}+2}$, find the value of $x+\frac{1}{x}$.

Q.24 Express the following in the form of p/q.

(i) $.\overline{3}$ (ii) $.\overline{54}$ (iii) $1.\overline{3}$ (iv) $0.\overline{621}$

Q.25 Find two irrational numbers between 0.5 and 0.55.

Q.26 Write two irrational numbers between 0.2 and 0.21.

Q.27 Write three irrational numbers between 0.20200200020002.... and 0.2030030003000300003..... .

Q.28 (i) Write two irrational numbers between 0.21 and 0.2222.....

(ii) Find three different irrational numbers between the rational numebrs $\frac{5}{9}$ and $\frac{5}{7}$

Q.29 Write three irrational numebrs between $\sqrt{3}$ and $\sqrt{5}$.

Q.30 Represent $\sqrt{8.3}$ on the number line.

Q.31 Find the rationalising factor of -

(i) $\sqrt[3]{49}$ (ii) $\sqrt[4]{5}$ (iii) $\sqrt{7}+\sqrt{3}$

Q.32 If $x = 3 - 2\sqrt{2}$, find the value of

(i) $\frac{1}{x}$ (ii) $x+\frac{1}{x}$ (iii) $x-\frac{1}{x}$

(iv) $\sqrt{x}+\frac{1}{\sqrt{x}}$ (v) $\sqrt{x}-\frac{1}{\sqrt{x}}$

(vi) $x^2+\frac{1}{x^2}$ (vii) $x^2-\frac{1}{x^2}$

(viii) $x^3+\frac{1}{x^3}$ (ix) $x^3-\frac{1}{x^3}$

(x) $x^4+\frac{1}{x^4}$ (xi) $x^4-\frac{1}{x^4}$

Q.33 In each of the following express the result in the simplest form.

(i) $\sqrt[3]{-108a^4b^3}$ (ii) $\sqrt[8]{512}$

(iii) $\sqrt[4]{a^8b^6c^7}$ (iv) $\sqrt[3]{\frac{27}{a^5b^6c^4}}$

(v) $2.\sqrt[3]{40}+3.\sqrt[3]{625}+4.\sqrt[3]{320}$

(vi) $8.\sqrt{242}-5.\sqrt{50}+3.\sqrt{98}$

Q.34 Simplify

(i) $\frac{\sqrt[4]{3888}}{\sqrt[4]{48}}$

(ii) $3\sqrt[4]{\sqrt[3]{5}}-\sqrt[3]{\sqrt[4]{5}}$

(iii) $\sqrt[5]{\sqrt[4]{(2^4)^3}}-5\sqrt[5]{8}+2\sqrt[5]{\sqrt[4]{(2^3)^4}}$

Q.35 Find value of x :

(i) $\sqrt[4]{3x+1}=2$ (ii) $\sqrt[3]{4x-7}-5=0$

(iii) $\sqrt[24]{49}=(x)^{1/12}$ (iv) $\sqrt[3]{2}=(x)^{1/12}$

Q.36 Given that $\sqrt{3}=1.732$, find the value of

$\sqrt{75}+\frac{1}{2}\sqrt{48}-\sqrt{192}$

Q.37 If x = $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ and $y=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$, fidn the value of $x^2 + y^2 - 6xy$.

Q.38 If $x=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $y=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, find the value of $x^2 + y^2 + xy$.

Q.39 Show that : $\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}$ $-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}=5$

Q.40 Simplify : $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}-\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$

Q.41 Simplify:

$$\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{a-c}+x^{b-c}}$$

Q.42 Prove that $\frac{a^{-1}}{a^{-1}+b^{-1}}+\frac{a^{-1}}{a^{-1}-b^{-1}}=\frac{2b^2}{b^2-a^2}$

Q.43 If $\sqrt{2}=1.414, \sqrt{3}=1.732, \sqrt{5}=2.236$ and $\sqrt{6}=2.449$, find the value of $\frac{2+\sqrt{3}}{2-\sqrt{3}}+\frac{2-\sqrt{3}}{2+\sqrt{3}}+\frac{\sqrt{3}-1}{\sqrt{3}+1}$

Q.44 Express with a rational denominator :

(i) $\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$

(ii) $\frac{1}{\sqrt{10}+\sqrt{14}+\sqrt{15}+\sqrt{21}}$

Q.45 If xyz = 1, then show that

$(1 + x + y^{-1})^{-1} + (1 + y + z^{-1})^{-1} + (1 + z + x^{-1})^{-1} = 1$

Q.46 Find two irrational numbers lying between $\sqrt{2}$ and $\sqrt{3}$

Q.47 Show that 0.2353535... = $0.2\overline{35}$ can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$

Q.48 Express $0.2\overline{45}$ as a friction in simplest form

Q.49 Find a rational number and also an irrational number lying between the numbers 0.3030030003 ... and 0.3010010001 ....

ANALYTICAL QUESTIONS

BASIC LEVEL

Q.1 For an integer n, a student states the following:

I. If n is odd, $(n + 1)^2$ is even.

II. If n is even, $(n - 1)^2$ is odd.

III. If n is even, $\sqrt{(n-1)}$ is irrational.

Which of the above statements would be true?

(1) I and III (2) I and II

(3) I, II and III (4) II and III

Q.2 The sum of two rational numbers is .... number

(1) Rational (2) Natural

(3) Either (1) or (2) (4) Irrational

Q.3 Rational number between $\sqrt{2}$ and $\sqrt{3}$ is

(1) $\frac{\sqrt{2}+\sqrt{3}}{2}$ (2) $\frac{\sqrt{2}\times\sqrt{3}}{2}$

(3) 1.5 (4) 1.8

Q.4 Two irrational numbers between $\sqrt{2}$ and $\sqrt{3}$ are

(1) $2^{1/2}$, $6^{1/4}$ (2) $3^{1/4}$, $3^{1/6}$

(3) $6^{1/8}$, $3^{1/4}$ (4) $3^{1/2}$, $3^{1/8}$

Q.5 The rationalising factor of $\sqrt[5]{a^2b^3c^4}$ is

(1) $\sqrt[5]{a^3b^2c}$ (2) $\sqrt[4]{a^3b^2c}$

(3) $\sqrt[3]{a^3b^2c}$ (4) $\sqrt{a^3b^2c}$

Q.6 The rational number between 1/2 and 1/3 is

(1) 2/5 (2) 1/5 (3) 3/5 (4) 4/5

Q.7 Which of the following fractions lie between 1/5 and 1/4?

(A) $\frac{7}{33}$ (B) $\frac{4}{11}$ (C) $\frac{13}{57}$ (D) $\frac{7}{17}$

(1) A and B (2) A and C

(3) B, C and D (4) A, B and D

Q.8 Express $0.\overline{34}+0.3\overline{4}$ as a single decimal.

(1) $0.67\overline{88}$ (2) $0.6\overline{89}$

(3) $0.68\overline{78}$ (4) $0.6\overline{87}$

Q.9 If $\sqrt{5^n}$ = 125, then $5^{\sqrt[n]{64}}$ = __________.

(1) 25 (2) $\frac{1}{125}$

(3) 625 (4) $\frac{1}{25}$

Q.10 Which of the following pairs having two equal values? (where $x \in R$) _________.

(1) $9^{x/2}$, $27^{x/3}$ (2) $(256)^{4/x}$, $(4^3)^{4/x}$

(3) $(343)^{x/3}$, $(7^4)^{x/12}$ (4) $(36^2)^{2/7}$, $(6^3)^{2/7}$

Q.11 If $\sqrt{2^n}$ = 1024, then $3^{2\left(\frac{n}{4}-4\right)}$ = __________.

(1) 3 (2) 9 (3) 27 (4) 81

Q.12 $(\sqrt[6]{5})(\sqrt[3]{2})(\sqrt{3})(\sqrt[12]{6})$ =

(1) $\sqrt[12]{1749600}$ (2) $\sqrt[3]{2}\times\sqrt[12]{109350}$

(3) $\sqrt[12]{177960}$ (4) Both (1) and (2)

Q.13 What is the value of 4^{2x-2}, if $(16)^{2x+3} = (64)^{x+3}$?

(1) 64 (2) 256 (3) 32 (4) 512

Q.14 If m and n are positive integers, then for a positive number a, $\left\{\sqrt[m]{(\sqrt[n]{a})}\right\}^{mn}$ __________.

(1) a^{mn} (2) a (3) $a^{m/n}$ (4) 1

Q.15 Express the following in the simplest form $\frac{\sqrt[3]{81}}{\sqrt[3]{3}}$

(1) 9 (2) 3 (3) 27 (4) 81

Q.16 Express the surd $\frac{3}{\sqrt{11}}$ with rational denominator.

(1) $3\sqrt{11}$ (2) $9\sqrt{11}$

(3) $\frac{3\sqrt{11}}{11}$ (4) $\frac{9\sqrt{11}}{11}$

Q.17 $\sqrt{7+\sqrt{48}}$ = _________.

(1) $\sqrt{2}+\sqrt{3}$ (2) $\sqrt{3}+2\sqrt{2}$

(3) $2\sqrt{3}+\sqrt{2}$ (4) $2+\sqrt{3}$

Q.18 Simplify : $2\sqrt{12}-3\sqrt{32}+2\sqrt{48}$.

(1) $12\sqrt{3}-12\sqrt{2}$ (2) $12\sqrt{3}-8\sqrt{2}$

(3) $8\sqrt{3}-12\sqrt{2}$ (4) $\sqrt{3}-\sqrt{2}$

Q.19 If x = $\frac{2-\sqrt{3}}{2+\sqrt{3}}$, find the value of $x+\frac{1}{x}$.

(1) 7 (2) 21 (3) 14 (4) 1

Q.20 If x = $\frac{11}{4-\sqrt{5}}$, find the value of $x^2 - 8x + 11$.

(1) –1 (2) 0 (3) ±1 (4) 1

Q.21 Find the positive square root of the following :

$10+2\sqrt{6}+\sqrt{60}+2\sqrt{10}$

(1) $\sqrt{2}+\sqrt{3}-\sqrt{5}$ (2) $\sqrt{2}+\sqrt{3}+\sqrt{5}$

(3) $2\sqrt{2}+3\sqrt{3}+5\sqrt{5}$ (4) $\sqrt{2}+3\sqrt{3}+5$

Q.22 Simplify the following :

$$\frac{5\sqrt{5}}{\sqrt{11}+\sqrt{6}}-\frac{3\sqrt{3}}{\sqrt{6}+\sqrt{3}}-\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

(1) $\sqrt{55}-3\sqrt{2}-3$ (2) $\sqrt{55}-3\sqrt{3}-2$

(3) $\sqrt{55}-2\sqrt{3}-3$ (4) $-\sqrt{55}-2\sqrt{3}-3$

Q.23 Given $\sqrt{2}=1.414, \sqrt{3}=1.732,$ find the value, correct to three decimals, of the following $\frac{1-\sqrt{3}}{\sqrt{5}-\sqrt{2}}$:

(1) – 0.0891 (2) – 0.8091

(3) – 0.891 (4) – 8.91

Q.24 If $x=3\sqrt{3}+\sqrt{26}$, find the value of $\frac{1}{2}\left(x+\frac{1}{x}\right)$.

(1) $3\sqrt{3}$ (2) $2\sqrt{3}$ (3) $2\sqrt{2}$ (4) $9\sqrt{3}$

Q.25 If $x = \frac{1}{7+4\sqrt{3}}, y=\frac{1}{7-4\sqrt{3}}$, find the value of $5x^2 - 7xy - 5y^2$.

(1) $\frac{-7\sqrt{2}+24\sqrt{3}+9\sqrt{6}-30}{92}$

(2) $\frac{-7\sqrt{2}-24\sqrt{3}+9\sqrt{6}+30}{92}$

(3) $\frac{+7\sqrt{2}+24\sqrt{3}+9\sqrt{6}+30}{92}$

(4) $\frac{-7\sqrt{2}-24\sqrt{3}-9\sqrt{6}-30}{92}$

Q.26 Rationalize the denominator of the following:

$$\frac{\sqrt{2}+3\sqrt{5}}{3\sqrt{7}+5\sqrt{3}}$$

(1) $\frac{3\sqrt{14}+9\sqrt{35}+5\sqrt{6}+15\sqrt{15}}{-12}$

(2) $\frac{3\sqrt{14}+9\sqrt{35}-5\sqrt{6}-15\sqrt{15}}{-12}$

(3) $\frac{3\sqrt{14}-9\sqrt{35}-5\sqrt{6}-15\sqrt{15}}{-12}$

(4) $\frac{-3\sqrt{14}-9\sqrt{35}+5\sqrt{6}+15\sqrt{15}}{-12}$

Q.27 Given $\sqrt{3}=1.7321$, find the value of the following surd, correct to three decimal places.

$$\frac{\sqrt{3}+1}{\sqrt{3}+1}+\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{4+\sqrt{3}}{4-\sqrt{3}}$$

(1) 6.527

(2) 6.257

(3) 6.725

(4) 6.275

Q.28 Simplify :

Column-I		Column-II	
(a)	$\left(\frac{81}{16}\right)^{-\frac{3}{4}} \times \left\{\left(\frac{25}{9}\right)^{-3/2} \div \left(\frac{5}{2}\right)^{-3}\right\}$	(p)	$\frac{3}{80}$
(b)	$\frac{\sqrt[3]{0.125}\times\sqrt[5]{(0.00032)^{-2}}}{\sqrt[5]{(0.00243)^{-3}\times(27)^{2/3}}}$	(q)	$\frac{39+8\sqrt{30}}{21}$
(c)	$\sqrt[4]{(81)^{-2}}$	(r)	1/9
(d)	$\frac{2\sqrt{6}+\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$	(s)	1

(1) (a) → (s), (b) → (p), (c) → (r), (d) → (q)

(2) (a) → (p), (b) → (q), (c) → (r), (d) → (s)

(3) (a) → (r), (b) → (p), (c) → (q), (d) → (s)

(4) (a) → (q), (b) → (r), (c) → (p), (d) → (s)

Q.29 Evaluate the following if $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{2}$ and π = 3.141 upto three places of decimals.

	Column-I		Column-II
(a)	$\frac{2}{\sqrt{5}-\sqrt{3}}$	(p)	4.357
(b)	$\frac{\pi}{2}+\frac{3}{\sqrt{5}}$	(q)	3.968
(c)	$\frac{1}{2\sqrt{5}-3\sqrt{2}}$	(r)	2.912
(d)	$\pi+\frac{1}{\sqrt{2}}$	(s)	3.848

(1) (a) $\rightarrow$ (q), (b) $\rightarrow$ (r), (c) $\rightarrow$ (p), (d) $\rightarrow$ (s)
(2) (a) $\rightarrow$ (p), (b) $\rightarrow$ (q), (c) $\rightarrow$ (r), (d) $\rightarrow$ (s)
(3) (a) $\rightarrow$ (r), (b) $\rightarrow$ (p), (c) $\rightarrow$ (q), (d) $\rightarrow$ (s)
(4) (a) $\rightarrow$ (s), (b) $\rightarrow$ (r), (c) $\rightarrow$ (p), (d) $\rightarrow$ (q)

Q.30 The number of ordered pairs (a, b) of positive integers such that a + b = 90 and their greatest common divisor is 6 equals.
(1) 5 (2) 4 (3) 14 (4) 10

Q.31 Which is the smallest six-digit number divisible by 111?
(1) 111111 (2) 110011
(3) 100011 (4) 100111

Q.32 Number of values of 'a' (from 0 to 9) for the number N = 2345631143a4 is divisible by 12, is :
(1) 0 (2) 1
(3) 2 (4) more than 2

Q.33 $1^{13} + 2^{13} + 3^{13} + ... + 60^{13}$ is divisible by :
(1) 61 (2) 63 (3) 65 (4) 59

Q.34 Which of the following number is divisible by 99?
(1) 3572404 (2) 135792
(3) 913464 (4) 114345

Q.35 There is an N digit number (N > 1). If the sum of digits is subtracted from the number then the resulting number will be divisible by:
(1) 7 (2) 2 (3) 11 (4) 9

Q.36 The smallest integral value of x, for which 7/x is an integer is:
(1) 1 (2) – 1 (3) 7 (4) – 7

Q.37 The smallest prime number that divides the sum $(7^{11} + 11^{13})$ is :
(1) 2 (2) 3 (3) 5 (4) 7

Q.38 If x is a positive integer such that 2x + 12 is perfectly divisible by 'x', then the number of possible values of 'x' is :
(1) 2 (2) 5 (3) 6 (4) 12

Q.39 The least number which on division by 35 leaves a remainder 25 and on division by 45 leaves the remainder 35 and on division by 55 leaves the remainder 45 is :
(1) 2515 (2) 3455 (3) 2875 (4) 2785

Q.40 Which one of the numbers listed below is not a divisor of the number N = $(2^{30} - 1)$, is equal to :
(1) $2^5 - 1$ (2) $2^5 + 1$ (3) $2^6 - 1$ (4) $2^{10} - 1$

Q.41 The remainder when $(x^{51} + 51)$ is divided by (x + 1), is :
(1) 0 (2) 1 (3) 51 (4) 50

Q.42 A number divided by 14 gives a remainder 8. What is the remainder, if this number is divided by 7?
(1) 1 (2) 2 (3) 3 (4) 4

Q.43 When $4^{101} + 6^{101}$ is divided by 25, the remainder is :
(1) 20 (2) 10 (3) 5 (4) 0

Q.44 What is the remainder when 13^{400} is divided by 11?
(1) –1 (2) 1 (3) 5 (4) 2

Q.45 A number n is said to be perfect if the sum of all its divisors (excluding n itself) is equal to n. An example of perfect number is
(1) 6 (2) 9 (3) 15 (4) 21

Q.46 If the product of two irrational numbers is rational, then which of the following can be concluded?
(1) The ratio of the greater and the smaller numbers is an integer
(2) The sum of the numbers must be rational
(3) The excess of the greater irrational number over the smaller irrational number must be rational
(4) None of these

Q.47 Which pair of numbers below are twin primes?
(1) 8 and 9 (2) 2 and 3
(3) 3 and 7 (4) 41 and 43

Q.48 Find the digit in the units place of $(676)^{99}$.

(1) 9 (2) 2 (3) 4 (4) 6

Q.49 Which of the following is not a rational number ?

(1) $\sqrt{2}$ (2) $\sqrt{4}$ (3) $\sqrt{9}$ (4) $\sqrt{16}$

Q.50 Of which of the following is the set of natural numbers a subset ?

(1) The set of even numbers
(2) The set of odd numbers
(3) The set of composite numbers
(4) The set of real numbers

Q.51 Which of an irrational number between $\sqrt{2}$ and $\sqrt{3}$?

(1) $2^{\frac{1}{2}}$ (2) $3^{\frac{1}{4}}$ (3) $6^{\frac{1}{4}}$ (4) $6^{\frac{1}{8}}$

Q.52 What type of a number is $\left(6+\sqrt{2}\right)\left(6-\sqrt{2}\right)$?

(1) Rational number (2) Irrational number
(3) Prime number (4) Negative Integer

Q.53 What type of a number is $\left(\sqrt{2}+\sqrt{3}\right)^2$?

(1) A rational number
(2) An irrational number
(3) A friction
(4) A decimal number

Q.54 Given X : Every fraction is a rational number and Y : Every rational number is fraction.
Which of the following is correct ?

(1) X is False and Y is True
(2) X is True and Y is false
(3) Both X and Y are true
(4) Both X and Y are false

Q.55 What is the rationalizing factor of $\left(2\sqrt[3]{5}\right)$?

(1) $\sqrt[3]{5}$ (2) $\sqrt[3]{5^2}$ (3) 5^2 (4) 5^3

Q.56 What is the rationalizing factor of $\sqrt[5]{a^2b^3c^4}$?

(1) $\sqrt[5]{a^3b^2c}$ (2) $\sqrt[4]{a^3b^2c}$
(3) $\sqrt[3]{a^3b^2c}$ (4) $\sqrt{a^3b^2c}$

Q.57 What is the rational denominator of $\frac{2\sqrt[3]{5}}{\sqrt[3]{9}}$?

(1) 1 (2) 2 (3) 3 (4) 4

Q.58 Find the value of $\left(\sqrt[6]{27}-\sqrt{6\frac{3}{4}}\right)^2$

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{3}{2}$ (3) $\frac{\sqrt{3}}{4}$ (4) $\frac{3}{4}$

Q.59 What is the product of $\sqrt[3]{4}$ and $\sqrt[3]{22}$?

(1) $2\sqrt[3]{11}$ (2) $\left(2\sqrt{11}\right)$
(3) $\left(2\sqrt{11}\right)$ (4) $8\sqrt[3]{11}$

Q.60 What is the quotient when $\sqrt[6]{12}$ is divided by $\sqrt{3}\,\sqrt[3]{2}$?

(1) $\frac{1}{\sqrt[2]{3}}$ (2) $\frac{1}{\sqrt[3]{3}}$ (3) $\frac{1}{\sqrt[4]{3}}$ (4) $\frac{1}{\sqrt[5]{3}}$

Q.61 If $\frac{-4}{7}=\frac{-32}{x}$, what is the value of x ?

(1) – 56 (2) 56 (3) – 65 (4) 65

Q.62 Find the value of $x - y^{x-y}$ when x = 2 and y = – 2.

(1) 18 (2) – 18 (3) 14 (4) – 14

Q.63 If $9^{x+2} = 240 + 9^x$, find x.

(1) 0.5 (2) 0.2 (3) 0.4 (4) 0.1

Q.64 Which expression is equal to $(x^{-1} + y^{-1})^{-1}$?

(1) xy (2) x + y (3) $\frac{xy}{x+y}$ (4) $\frac{x+y}{xy}$

Q.65 If $10^x = 64$, what is the value of $10^{\frac{x}{2}+1}$?

(1) 18 (2) 42 (3) 80 (4) 81

Q.66 If $\frac{x}{x^{1.5}}=8x^{-1}$ and x > 0, find x.

(1) $\frac{\sqrt{2}}{4}$ (2) $2\sqrt{2}$ (3) 4 (4) 64

Q.67 If $4^x - 4^{x-1} = 24$, what is the value of $(2x)^x$?

(1) $5\sqrt{5}$ (2) $\sqrt{5}$ (3) $25\sqrt{5}$ (4) 125

Q.68 If $x=\sqrt{5}+2$, find the value of $x-\frac{1}{x}$

(1) $2\sqrt{5}$ (2) 4 (3) 2 (4) $\sqrt{5}$

Q.69 If $x=\frac{2}{3-\sqrt{7}}$, find the value of $(x - 3)^2$.

(1) 1 (2) 3 (3) 6 (4) 7

Q.70 If $x = 7 + 4\sqrt{3}$ and xy = 1, what is the value of $\frac{1}{x^2}+\frac{1}{y^2}$?

(1) 64 (2) 134 (3) 194 (4) 1/49

Q.71 If $\sqrt{5}=2.236$ $\sqrt{3}=1.732$, find the value of $\frac{1}{\sqrt{5}-\sqrt{3}}$

(1) 3.968 (2) $\frac{1}{3.968}$ (3) 1.984 (4) $\sqrt{0.504}$

Q.72 Identify a rational number between –5 and 5.
(1) 0 (2) –7.3
(3) –5.7 (4) 1.101100110001.

Q.73 The sum of the digits a number is subtracted from the number, the resulting number is always divisible by which of the following numbers ?
(1) 2 (2) 5 (3) 8 (4) 9

Q.74 Find the difference between $1.\bar{4}$ and $1.\bar{2}$.

(1) $1.\bar{2}$ (2) $1.\bar{3}$ (3) $\frac{11}{9}$ (4) $\frac{12}{9}$

Q.75 If $\sqrt{a}$ is an irrational number, what is a ?
(1) Rational (2) Irrational
(3) 0 (4) Real

Q.76 Which of the following is an irrational number ?
(1) $\sqrt{9}$ (2) π
(3) $\frac{22}{7}$ (4) 2.5151515...

Q.77 Which of the following numbers is an irrational number ?
(1) $\sqrt{16}-4$ (2) $\left(3-\sqrt{3}\right)\left(3+\sqrt{3}\right)$
(3) $\sqrt{5}+3$ (4) $-\sqrt{25}$

Q.78 If $x = 2 + 2^{1/3} + 2^{2/3}$, find the value of $(x^3 - 6x^2 + 6x)$
(1) 2 (2) 4 (3) 8 (4) 6

Q.79 If $x=\frac{1}{2-\sqrt{3}}$, find the value of $x^3 - 2x^2 - 7x + 10$
(1) 14 (2) 8 (3) 4 (4) 16

Q.80 If a = 3 and b = 5, find the value of $a^a + b^b$.
(1) 512 (2) 251 (3) 513 (4) 152

Q.81 If p = 3 and q = 5, find the value of $\left(\frac{1}{p}+\frac{1}{q}\right)^p$

(1) $\frac{3375}{512}$ (2) $\frac{512}{3375}$ (3) $\frac{512}{225}$ (4) $\frac{64}{225}$

Q.82 Find the value of $\frac{1}{\sqrt{5}-\sqrt{4}}$

(1) $\sqrt{5}-\sqrt{4}$ (2) $\sqrt{5}+\sqrt{4}$
(3) $\frac{\sqrt{5}+\sqrt{4}}{9}$ (4) $\sqrt{4}-\sqrt{5}$

Q.83 Find the rational number for $0.\bar{5}$.

(1) $\frac{9}{5}$ (2) $\frac{5}{10}$ (3) $\frac{5}{9}$ (4) $\frac{10}{5}$

Q.84 Identify the simplest form of $\frac{5\sqrt{3}+3\sqrt{5}}{5\sqrt{3}-3\sqrt{5}}$

(1) $6+\sqrt{15}$ (2) $4+\sqrt{15}$
(3) $4+\sqrt{3}$ (4) $6+\sqrt{5}$

Q.85 Find the value of $\frac{(0.6)^0-(0.1)^{-1}}{\left(\frac{3}{8}\right)^{-1}\left(\frac{3}{2}\right)^3+\left(\frac{-1}{3}\right)^{-1}}$

(1) $\frac{-2}{3}$ (2) $\frac{3}{2}$ (3) $-\frac{3}{2}$ (4) $\frac{2}{3}$

Q.86 m and n are integers and $\sqrt{mn}=10$. Which of the following connot be a value of m + n ?
(1) 25 (2) 52 (3) 101 (4) 50

Q.87 If 2.5252525 .... $=\frac{p}{q}$ (in the lowest form) what is the value of $\frac{q}{p}$

(1) 0.4 (2) 0.42525
(3) 0.0396 (4) 0.396

Q.88 If $x^{x\sqrt{x}}=\left(x\sqrt{x}\right)^x$, find the value of x.

(1) $\frac{3}{2}$ (2) $\frac{2}{3}$ (3) $\frac{9}{4}$ (4) $\frac{4}{9}$

Q.89 Simplify $\frac{a^{\frac{1}{2}}+a^{-\frac{1}{2}}}{1-a}+\frac{1-a^{-\frac{1}{2}}}{1+\sqrt{a}}$

(1) 1 (2) 0 (3) $\frac{2}{1-a}$ (4) 1 + a

Q.90 Which symbol is used to denote a collection of all positive integers ?
(1) N (2) W (3) Z (4) Q

Q.91 Find the value of 0.9999 .... in the form of $\frac{p}{q}(p,q\in Z \text{ and } q\neq 0)$

(1) $\frac{1}{9}$ (2) $\frac{2}{9}$ (3) $\frac{9}{10}$ (4) 1

Q.92 If $x=\sqrt[3]{2+\sqrt{3}}$, find the value of $x^3+\frac{1}{x^3}$

(1) 2 (2) 4 (3) 8 (4) 9

Q.93 Find the simplest rationalising factor of $5^{1/3} + 5^{-1/3}$.
(1) $5^{2/3} + 1 + 5^{-2/3}$ (2) $5^{1/3} - 5^{-1/3}$
(3) $5^{2/3} - 1 + 5^{-2/3}$ (4) $5^{2/3} + 1 - 5^{-2/3}$

ADVANCE LEVEL

Q.1 Simplify

$$\frac{1}{\sqrt{19-\sqrt{360}}}-\frac{1}{\sqrt{21-\sqrt{440}}}+\frac{2}{\sqrt{20-\sqrt{396}}}$$

(1) 1 (2) 2
(3) 0 (4) None of these

Q.2 $\left(\sqrt[6]{5}\right)\left(\sqrt[3]{2}\right)\left(\sqrt{3}\right)\left(\sqrt[12]{6}\right)=$

(1) $\sqrt[12]{1749600}$ (2) $\sqrt[3]{2}\times\sqrt[12]{109350}$
(3) $\sqrt[12]{177960}$ (4) Both (1) & (2)

Q.3 If x = $\sqrt{6}+\sqrt{5}$, then $x^2+\frac{1}{x^2}-2=$

(1) $2\sqrt{6}$ (2) $2\sqrt{5}$
(3) 24 (4) 20

Q.4 If $a^p = b^q = c^r = abc$, then pqr = _______.
(1) $p^2 q + q^2 r$ (2) pq + qr + pr
(3) $(pq + qr + rp)^2$ (4) pq (qr + rp)

Q.5 If $\sqrt{2^n}$ = 1024, then $3^{2\left(\frac{n}{4}-4\right)}$ = ______.
(1) 3^2 (2) –3 (3) 27 (4) 81

Q.6 The value of is equal to $\left(\frac{x^q}{x^r}\right)^{\frac{1}{qr}}\times\left(\frac{x^r}{x^p}\right)^{\frac{1}{rp}}\times\left(\frac{x^p}{x^q}\right)^{\frac{1}{pq}}$

(1) $x^{\frac{1}{p}+\frac{1}{q}+\frac{1}{r}}$ (2) 0
(3) $x^{pq+qr+rp}$ (4) 1

Q.7 The numerator of this

$$\frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}}+\frac{a-\sqrt{a^2-b^2}}{a+\sqrt{a^2-b^2}} \text{ is}$$

(1) a^2 (2) b^2
(3) $a^2 - b^2$ (4) $4a^2 - 2b^2$

Q.8 The value of

$$\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+.........+\frac{1}{99\times100} \text{ is}$$

(1) Less than $\frac{99}{100}$ (2) Equal to $\frac{99}{100}$
(3) Greater than $\frac{100}{99}$ (4) Equal to $\frac{100}{99}$

Q.9 If $25^{x-1} = 5^{2x-1} - 100$, then the value of x is
(1) 3 (2) 2 (3) 4 (4) 1

Q.10 Value of $\left[\left(x^{a-a^{-1}}\right)^{\frac{1}{a-1}}\right]^{\frac{a}{a+1}}$

(1) x (2) $\frac{1}{x}$ (3) x^a (4) $\frac{1}{x^a}$

Q.11 If p = $7-4\sqrt{3}$, then $\frac{p^2+1}{7p}$ = ______.
(1) 2 (2) 1 (3) 7 (4) $\sqrt{3}$

Q.12 If = $\sqrt[x]{3}\times\sqrt[y]{5}$ = 10125, then 12xy = ______.
(1) 1 (2) $\frac{1}{3}$ (2) 2 (4) $\frac{1}{2}$

Q.13 if $\frac{2^{m+n}}{2^{m-n}}$ =16 and a = $2^{\frac{1}{10}}$ then $\frac{\left(a^{2m+n-p}\right)^2}{\left(a^{m-2n+2p}\right)^{-1}}$ =___.

(1) 2 (2) $\frac{1}{4}$ (3) 9 (4) $\frac{1}{8}$

Q.14 Simplify

$$\left[\left(p^{-1}+q^{-1}\right)\left(p^{-1}-q^{-1}\right)\div\left(\frac{1}{p^{-1}}-\frac{1}{q^{-1}}\right)\left(\frac{1}{p^{-1}}+\frac{1}{q^{-1}}\right)\right](pq)^2$$

(1) $(pq)^2$ (2) $\left(\frac{-1}{pq}\right)$ (3) $-(pq)^{-2}$ (4) 1

Q.15 If x = $\frac{1}{5+2\sqrt{6}}$, then $x^2 - 10x + 1$= _______.
(1) 1 (2) –1 (3) 0 (4) 10

Q.16 What is the digit in the tens place in the product of the first 35 even natural numbers?
(1) 6 (2) 2 (3) 0 (4) 5

Q.17 Find the unit's digit in the product of the first 50 odd natural numbers.
(1) 0 (2) 5
(3) 7 (4) None of these

Q.18 What is the number in the units place of $(763)^{84}$?
(1) 1 (2) 3 (3) 7 (4) 9

Q.19 If the numbers a – b and a + b are twin primes, then a and b are necessarily
(1) Twin primes (2) Co-primes
(3) Cannot say (4) None

Q.20 Find the units digit in the expansion of $(44)^{44} + (55)^{55} + (88)^{88}$.
(1) 7 (2) 5 (3) 4 (4) 3

Q.21 If $x > 0$, $y > 0$ and $z > 0$ and $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}$ $(x + y + z)\,[(x-y)^2 + (y-z)^2 + (z-x)^2]$, then

(1) $x^3 + y^3 + z^3 \geq 3xyz$ (2) $x^3 + y^3 + z^3 \geq 6xyz$
(3) $x^3 + y^3 + z^3 > 3xyz$ (4) $x^3 + y^3 + z^3 < 3xyz$

Q.22 $2^{x-3} . 3^{2x-8} = 36$, then the value of x is _______.

(1) 2 (2) 5 (3) 3 (4) 1

Q.23 If $N = \dfrac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}} - \sqrt{3-2\sqrt{2}}$, then N equals ______.

(1) 1 (2) $2\sqrt{2}-1$ (3) $\dfrac{\sqrt{5}}{2}$ (4) $\dfrac{2}{\sqrt{\sqrt{5}+1}}$

Q.24 Express the mixed recurring decimal $4.3\overline{2}$ in the form $\frac{p}{q}$.

(1) $\dfrac{389}{90}$ (2) $\dfrac{329}{90}$ (3) $\dfrac{29}{90}$ (4) $\dfrac{233}{990}$

Q.25 If $x = (7 + 4\sqrt{3})$, then $(x + \frac{1}{x}) =$ ______.

(1) $8\sqrt{3}$ (2) 14 (3) 49 (4) 48

Q.26 $\dfrac{7\sqrt{3}}{\left(\sqrt{10}+\sqrt{3}\right)} - \dfrac{2\sqrt{5}}{\left(\sqrt{6}+\sqrt{5}\right)} - \dfrac{3\sqrt{2}}{\left(\sqrt{15}+3\sqrt{2}\right)} =$ ____.

(1) 1 (2) 2 (3) $\dfrac{1}{2}$ (4) 3

Q.27 The rationalising factor of $\sqrt[5]{a^2b^3c^4}$ is ______.

(1) $\sqrt[5]{a^3b^2c}$ (2) $\sqrt[4]{a^3b^2c}$
(3) $\sqrt[3]{a^3b^2c}$ (4) $\sqrt{a^3b^2c}$

Q.28 Rational number $\dfrac{-18}{5}$ lies between consecutive integers ________.

(1) –2 and –3 (2) –3 and –4
(3) –4 and –5 (4) –5 and –6

Q.29 An irrational number between $\dfrac{1}{7}$ and $\dfrac{2}{7}$ is _____.

(1) $\dfrac{1}{2}\left(\dfrac{1}{7}+\dfrac{2}{7}\right)$ (2) $\left(\dfrac{1}{7}\times\dfrac{2}{7}\right)$

(3) $\sqrt{\dfrac{1}{7}\times\dfrac{2}{7}}$ (4) None of these

Q.30 The greater number among $\sqrt{17}-\sqrt{12}$ and $\sqrt{11}-\sqrt{6}$.

(1) $\sqrt{17}-\sqrt{12}$ (2) $\sqrt{11}-\sqrt{6}$
(3) Both are equal (4) Can't be compared

Q.31 The value of x, if $5^{x-3} . 3^{2x-8} = 225$ is ___.

(1) 3 (2) 4 (3) 2 (4) 5

Q.32 The value of $\frac{1}{3}$ of 15^{27} is _____.

(1) 5^{27} (2) 15^{9}
(3) 5×15^{26} (4) 5×3^{9}

Q.33 If $x = 2 - \sqrt{3}$, then the value of $x^2 + \dfrac{1}{x^2}$ and $x^2 - \dfrac{1}{x^2}$ resrectively, are ____.

(1) $14,\ 8\sqrt{3}$ (2) $-14, -8\sqrt{3}$
(3) $14, -8\sqrt{3}$ (4) $-14, 8\sqrt{3}$

Q.34 Which of the following statements is INCORRECT?

(1) Every integer is a-rational number.
(2) Every natural number is an integer.
(3) Every natural number is real number.
(4) Every real number is a rational number.

Q.35 If $\dfrac{\left(\sqrt{3}-1\right)}{\left(\sqrt{3}+1\right)} = \left(a-b\sqrt{3}\right)$, find the value of a and b.

(1) a = 1, b = 2 (2) a = 2, b = 1
(3) a = 2, b = 3 (4) a = 3, b = 2

Q.36 Which of the following numbers has the terminating decimal representation ?

(1) $\dfrac{5}{12}$ (2) $\dfrac{8}{35}$ (3) $\dfrac{7}{24}$ (4) $\dfrac{13}{80}$

Q.37 The number x = 1.242424........ can be expressed in the form $x = \dfrac{p}{q}$, whre p and q are positive integers having no common factors.
Then p + q equals _______.

(1) 72 (2) 74 (3) 41 (4) 53

Q.38 If x and y are postive real numbers, then which of the following is CORRECT ?

(1) $x > y \Rightarrow -x > -y$ (2) $x > y \Rightarrow -x < -y$

(3) $x > y \Rightarrow \dfrac{1}{x} > \dfrac{1}{y}$ (4) $x > y \Rightarrow \dfrac{1}{x} < \dfrac{-1}{y}$

Q.39 If $x = 1 - \sqrt{2}$, then find the value of $\left(x-\dfrac{1}{x}\right)^2$

(1) 2 (2) 3 (3) 4 (4) 5

Q.40 The value of

$$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{5}}+$$

$$\frac{1}{\sqrt{5}+\sqrt{2}}+\frac{1}{\sqrt{6}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{9}}$$

is ______.

(1) 0

(2) 1

(3) 2

(4) 4

Q.41 Which of the following statements is INCORRECT?

(1) If 'a' is rational number and 'b' is irrational, then a + b is irrational.

(2) The product of a non - zero rational number with an irrationalnumbers is always irrational.

(3) Addition of any two irrational numbers can be rational.

(4) Division of any two integers is an integer.

Q.42 Fill in the blanks.

(i) The decimal form of an irrational number is neither P nor Q

(ii) There are R rational numbers between any two consecutive integers.

(iii) Every rational number is S.

(1) P-non- repeating, Q-terminating, R-zero, S-real

(2) P-repeating, Q-terminating, R-infinite, S-real

(3) P-non- repeating, Q-non terminating, R- zero infinite, S-integer

(4) P-repeating, Q-terminating, R- infinite, S-integer

Q.43 Read the statements carefully.

Statement-1: Every point on the number line represent a unique real number.

Statement 2: Irrational numbers cannot be represented on a number line.

Which of the following options hold ?

(1) Both Statement - 1and Statement - 2 are true.

(2) Statement - 1 is true but Statement -2 is false.

(3) Statement - 1 is false but Statement -2 is true.

(4) Both Statement - 1 and Statement - 2 are false.

PREVIOUS YEAR QUESTIONS

Q.1 If $2^x = 4^y = 8^z$ and $\frac{1}{2x}+\frac{1}{4y}+\frac{1}{6z}=\frac{24}{7}$, then the value of z is **[Raj. NTSE Stage-1 2005]**

(1) $\frac{7}{16}$ (2) $\frac{7}{32}$ (3) $\frac{7}{48}$ (4) $\frac{7}{64}$

Q.2 An equivalent expression of $\frac{5}{\sqrt{3}-\sqrt{5}}$ after rationalizing the denominator is

[Raj. NTSE Stage-1 2013]

(1) $\left(\frac{5}{2}\right)\left(\sqrt{3}+\sqrt{5}\right)$ (2) $\left(-\frac{5}{2}\right)\left(\sqrt{3}+\sqrt{5}\right)$

(3) $\left(\frac{5}{2}\right)\left(\sqrt{3}-\sqrt{5}\right)$ (4) $\left(-\frac{5}{2}\right)\left(\sqrt{3}-\sqrt{5}\right)$

Q.3 The simplest rationalizing factor of $3\sqrt{72}$ is

[M. P. NTSE Stage-1 2014]

(1) $2^{\frac{1}{3}}$ (2) $3^{\frac{1}{3}}$ (3) $3^{\frac{1}{2}}$ (4) $2^{\frac{1}{2}}$

Q.4 Simple form of $\frac{\sqrt{5}-2}{\sqrt{5}+2}+\frac{\sqrt{5}+2}{\sqrt{5}-2}$ is

[M. P. NTSE Stage-1 2015]

(1) $9+\sqrt{5}$ (2) 18

(3) $18+\sqrt{5}$ (4) 9

Q.5 The simplified value of

$$\frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}}+\frac{1}{\sqrt{2}-\sqrt{3}-\sqrt{5}} \text{ is}$$

[Delhi NTSE Stage-1 2015]

(1) 1 (2) 0 (3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

Q.6 An equivalent expression of $\frac{5}{7+4\sqrt{5}}$ after rationalizing the denominator is

[Gujarat NTSE Stage-1 2016]

(1) $\frac{20\sqrt{5}-35}{31}$ (2) $\frac{20\sqrt{5}-35}{129}$

(3) $\frac{35-20\sqrt{5}}{31}$ (4) $\frac{35-20\sqrt{5}}{121}$

Q.7 What is the value of $2.\bar{6}-1.\bar{9}$?

[Bihar NTSE Stage-1 2016]

(1) $0.\bar{6}$ (2) 0.9 (3) 0.7 (4) 0.7

Q.8 Read the following statements carefully and choose the correct alternative.

[Maharashtra NTSE STAGE-I 2018]

(A) The ratio of the circumference of a circle to its diameter is denoted by the Greek letter π

(B) π is non-terminating, recurring decimal fraction and its exact value is $\frac{22}{7}\left(\pi=\frac{22}{7}\right)$

Alternatives :

(1) Statements A and B false

(2) Statement A and B correct

(3) Statement A correct but B false

(4) Statement A false but B correct

Q.9 The value of $\frac{3}{4}+\frac{5}{36}+\frac{7}{144}+......+\frac{17}{5184}+\frac{19}{8100}$ is **[Bihar NTSE Stage-1 2019]**

(1) 0.95 (2) 0.98 (3) 0.99 (4) 1

ANSWER KEY

SUBJECTIVE QUESTIONS

Q.1 $11\sqrt{2}$ **Q.2** $9\sqrt{3}$ **Q.3** $30\sqrt{6}$

Q.4 6/7

Q.5 (i) $\frac{5}{8}$ = 0.625. This is a terminating decimal.

(ii) $\frac{22}{3}$ = 7.333333...... This is a recurring decimal

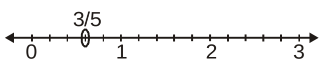
(iii) 12/7 = 0.142857142857142857............ This is a recurring decimal.

Q.6 (you knoow that there are actually infinitely many rational numbers between 3 and 4)

Q.7 (i) 4.1258673758 .....

(ii) 4.342157913458.....

(iii) 4.615857342168 ......

Q.8 

Q.9 $\frac{31}{80}$

Q.10 $\frac{5}{3};\frac{9}{14};\frac{5}{7};\frac{6}{11}$

Q.11 0.2751234......

Q.12 0.0500500050005.....; 5.51551555151....,

Q.13 $\frac{376}{333}$

Q.14 $5\sqrt{2}$

Q.15 $\sqrt{10}-\sqrt{5}$

Q.16 -2

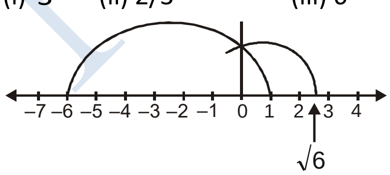
Q.17 12

Q.18 (i) $6+3\sqrt{3}+2\sqrt{2}+\sqrt{6}$ (ii) 22

(iii) $57+28\sqrt{2}$ (iv) $2(4-\sqrt{15})$

Q.19 (i) 3 (ii) 4 (iii) 8 (iv) 9

Q.20 (i) $3^{3/5}$ (ii) 2/3 (iii) $6^{2/3}$ (iv) 2

Q.21 

Q.22 2.32156..... and 2.36101.........

Q.23 18

Q.24 (i) 1/3 (ii) 6/11 (iii) 4/3 (iv) 23/37

Q.25 0.5010010001.....,0.5020020002....

Q.26 0.2010010001...., 0.2020020002....

Q.27 0.20201001000100001..., 0.202020020002..., 0.202030030003...

Q.28 (i) 0.21010010001...., 0.21020020002...

(ii) 0.6010010001....., 0.6020020002.......

Q.29 1.8010010001....., 1.9010010001...., 2.010010001.......

Q.31 (i) $\sqrt[3]{7}$ (ii) $\sqrt[4]{125}$ (iii) $\sqrt{7}-\sqrt{3}$

Q.32 (i) $3+2\sqrt{2}$ (ii) 6 (iii) $-4\sqrt{2}$

(iv) $\pm2\sqrt{2}$ (v) ±2 (vi) 34

(vii) $-24\sqrt{2}$ (viii) 198 (ix) $-140\sqrt{2}$

(x) 1154 (xi) $-816\sqrt{2}$

Q.33 (i) $-3ab.\sqrt[3]{4a}$ (ii) $2\sqrt[8]{2}$

(iii) $a^2bc\sqrt[4]{b^2c^3}$ (iv) $\frac{3}{ab^2c}\sqrt[3]{\frac{1}{a^2c}}$

(v) $35\sqrt[3]{5}$ (vi) $84\sqrt{2}$

Q.34 (i) 3 (ii) $2.\sqrt[12]{5}$ (iii) $-2.\sqrt[5]{8}$

Q.35 (i) 5 (ii) 33 (iii) 7

(iv) 16

Q.36 – 1.732

Q.37 56

Q.38 99

Q.40 1

Q.41 1

Q.43 14.268

Q.44 (i) $\sqrt{10}+\sqrt{5}$ (ii) $\frac{\sqrt{21}+\sqrt{10}-\sqrt{14}-\sqrt{15}}{2}$

ANALYTICAL QUESTIONS

BASIC LEVEL

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	2	3	3	3	1	1	2	4	1	1	2	4	2	2	2
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	3	4	1	3	2	2	1	3	1	4	2	1	1	1	3
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**
Ans.	3	3	1	4	4	4	1	3	2	4	4	1	2	2	1
Que.	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Ans.	4	4	4	1	4	3	1	2	2	2	1	3	4	1	2
Que.	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
Ans.	2	4	1	3	3	4	3	2	4	3	3	1	4	3	1
Que.	**76**	**77**	**78**	**79**	**80**	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**
Ans.	2	3	1	2	4	2	2	3	2	3	4	4	2	3	1
Que.	**91**	**92**	**93**												
Ans.	4	2	3												

ADVANCE LEVEL

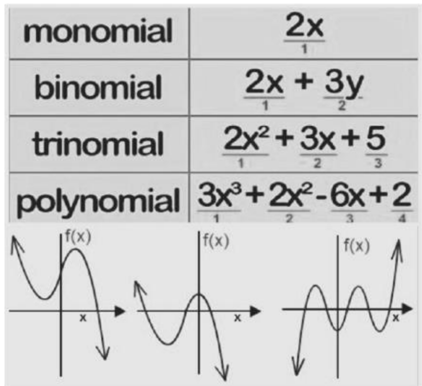
Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	3	4	4	2	2	4	3	2	2	1	1	1	1	2	3
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	3	2	1	2	1	1	2	1	1	2	1	1	2	3	2
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**		
Ans.	4	3	3	4	2	4	2	2	3	3	4	2	2		

PREVIOUS YEAR QUESTIONS

Que.	1	2	3	4	5	6	7	8	9
Ans.	3	2	2	2	4	1	1	3	3

Chapter-02

Polynomial



A polynomial is a mathematical expression consisting of variables, coefficients, and the operations of addition, subtraction, multiplication, and non-negative integer exponents.

2.1 INTRODUCTION

Before learning the meaning and scope of polynomials, the terms like constants, variables, algebraic expressions etc., have to be understood.

- **Constant**

  A number having a fixed numerical value is called a constant. **Ex.** $7, \frac{1}{2}, 4.7, 16.5$, etc.

- **Variable**

  A number which can take various numerical values is known as variable. **Ex.** x, y, z, a, b, c, etc.

  A variable raised to any non-zero real number is also a variable. **Ex.** x^5, $y^{\frac{10}{3}}$, $z^{0.9}$, etc.

  A number which is the product of a constant and a variable is also a variable. **Ex.** $8x^3, -7x^5, 4x^{10}$, etc.

  A combination of two or more variables separated by a (+) sign or a (–) sign is also a variable.

2.1.1 Algebraic Expression

A combination of constants and variables connected by +, –, × and signs is known as an algebraic expression. **Ex.** $8x + 7$, $11x^2 - 13x$, $5x^5 + 8x^2y$, etc.

- **Terms**

  The parts of an algebraic expression separated by + or – signs are called the terms of the expression.

  Ex. In the expression $3x + 4y - 7$, we call $3x$, $4y$ and -7 as terms.

- **Coefficient of a Term**

  Consider the term $8x^2$. In this case, 8 is called the numerical coefficient and x^2 is said to be the literal coefficient. In case of 9xy, we have the numerical coefficient as 9 and the literal coefficient as xy.

- **Like Terms**

  Terms having the same literal coefficients are called like terms.

  Ex. 1. $15x^2$, $-19x^2$ and $35x^2$ are all like terms. **2.** $8x^2y$, $5x^2y$ and $-7x^2y$ are all like terms.

- **Unlike Terms**

  Terms having different literal coefficients are called unlike terms. **Ex.** $5x^2$, $-10x$ and $15x^3$ are unlike terms.

2.1.2 Polynomial

An algebraic expression in which the variables involved have only non-negative integral powers is called a polynomial.

Ex. $5x^2 - 8x + 7$, $3x^3 + 5x^2 - 9$, $3y^2 - 5y + z$, etc.

The expression $3x^5 - 8x + \frac{4}{x} + 11x^{\frac{5}{2}}$ is not a polynomial. Since the exponents of x are negative integers and fractions.

Polynomial is an algebric expression of the form

$P(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots\ldots\ldots\ldots + a_2x^2 + a_1x^1 + a_0x^0$

Here $\rightarrow a_nx^n, a_{n-1}x^{n-1}, a_{n-2}x^{n-2} \rightarrow$ Terms of polynomial

a_n, a_{n-1}, a_{n-2} .... $\rightarrow$ Coefficients of terms.

$\rightarrow a_n \rightarrow$ leading coefficient ; $a_n \neq 0$

$\rightarrow n \rightarrow$ Degree of polynomial (Positive Integer)

A polynomial with one variable is known as a polynomial in that variable.

Ex. $5x^4 + 7x^3 + 3x - 9$ is a polynomial in the variable x.

$3y^3 + y^2 + y$ is a polynomial in the variable y.

$4x^2y^2 + 3xy^2 - 7xy$ is a polynomial in variable x and y.

2.2 DEGREE OF A POLYNOMIAL IN ONE VARIABLE

Degree of a polynomial terms is the sum of exponents of the variables that are appear on it

Degree of the polynomial is highest Among all polynomial terms.

Ex. $P(x) = x^2y^2 + x^3y + 5$

Degree of P(x) is '4'

The highest index of the variable in a polynomial of one variable is called the degree of the polynomial.

Ex. **1.** $11x^3 - 7x^2 + 5x + 2$ is a polynomial of degree 3.

2. $15x^6 - 8x + 7$ is a polynomial of degree 6.

2.2.1 Types of Polynomials with Respect to Degree

1. Linear polynomial: A polynomial of degree one is called a linear polynomial.

Ex. $11x - 5$, $10y + 7$ and $13z + 4$ are polynomials of degree one and hence they can be called as linear polynomials.

2. Quadratic polynomial: A polynomial of degree two is called a quadratic polynomial.

Ex. $5x^2 - 8x + 3$ and $13y^2 - 8y + 3$ are polynomials of degree two and hence can be called as quadratic polynomials.

3. Cubic polynomial: A polynomial of degree three is called a cubic polynomial.

Ex. $5x^3 + 6x^2 + 7x + 8$ and $4y^3 - 9y^2 + 3$ are polynomials of degree three and hence can be called as cubic polynomials.

4. Biquadratic polynomial: A polynomial of degree four is called a biquadratic polynomial.

Ex. $3x^4 - x^3 + 7x^2 - 2x + 1$ and $5x^4 - 2x + 7$ are polynomials of degree four and hence can be called as biquadratic polynomials.

5. Constant polynomial: A polynomial having only one term which is a constant is called a constant polynomial. Degree of a constant polynomial is 0.

Ex. 10, –11 are constant polynomials.

Special Case

If p(x) = 0, then it is called the **zero polynomial**. But the degree of zero polynomial is not defined, as p(x) can be written as

$p(x) = 0 = 0.x = 0.x^2 = 0.x^3 = \ldots\ldots$

2.2.2 Types of Polynomials with Respect to Number of Terms

1. Monomial: An expression containing only one term is called a monomial. **Ex.** $8x$, $-11x^2y$, $-15x^2y^3z^2$, etc.

2. Binomial: An expression containing two terms is called a binomial. **Ex.** $3x - 8y$, $4xy - 5x$, $9x + 5x^2$, etc.

3. Trinomial: An expression containing three terms is called a trinomial. **Ex.** $5x - 2y + 3z$, $x^2 + 2xy - 5z$, etc.

- **Addition of Polynomials**
- **Division of a Polynomial by a Polynomial**

Factor Method

In this method, we factorize the polynomial to be divided to that one or more of the factors is equal to the polynomial by which we wish to divide.

EX.1 : Divide $4x^2 + 7x - 15$ by $x + 3$.

Sol. $4x^2 + 7x - 15 = 4x^2 + 12x - 5x - 15$

$4x(x+3) - 5(x+3) = (4x-5)(x+3)$

$$\therefore \frac{4x^2+7x-15}{x+3} = \frac{(4x-5)(x+3)}{x+3} = 4x-5$$

- **Standard Formulae**

(i) $(a + b)^2 = a^2 + 2ab + b^2$

(ii) $(a - b)^2 = a^2 - 2ab + b^2$

(iii) $(a + b)(a - b) = a^2 - b^2$

(iv) $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

(v) $(a + b)^2 - (a - b)^2 = 4ab$

(vi) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = \Sigma a^2 + 2\Sigma ab$

(vii) $(a + b)^3 = a^3 + b^3 + 3ab(a + b) = a^3 + 3a^2b + 3ab^2 + b^3$

(viii) $(a - b)^3 = a^3 - b^3 - 3ab(a - b) = a^3 - 3a^2b + 3ab^2 - b^3$

(ix) $(x + a)(x + b) = x^2 + (a + b)x + ab$

(x) $(x + a)(x + b)(x + c) = x^3 + x^2(a + b + c) + x(ab + bc + ac) + abc$

(xi) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(xii) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(xiii) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

(xiv) If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

(xv) $a^2 + b^2 + c^2 - ab - bc - ca = 1/2\,[(a - b)^2 + (b - c)^2 + (c - a)^2]$

2.2.3 Value of A Polynomial

The value of a polynomial obtained on putting a particular value of the variable is called the value of a polynomial. The value of a polynomial p(x) at x = a(say) is denoted by p(a).

Ex. Let $p(x) = 5x^3 - 2x^2 + 3x - 2$

At x = 1, $p(1) = 5(1)^3 - 2(1)^2 + 3(1) - 2 = 5 - 2 + 3 - 2 = 8 - 4 = 4$

So, 4 is the value of given polynomial p(x) at x = 1.

2.2.4 Zero of A Polynomial

Zero of a polynomial $p(\alpha)$ is a number a, such that $p(\alpha) = 0$. Zero of a polynomial is also called the root of polynomial equation p(x) = 0. **Ex.** Let p(x) = 5x + 7.

At $x = \frac{-7}{5}, p\left(\frac{-7}{5}\right) = 5\left(\frac{-7}{5}\right) + 7 = -7 + 7 = 0$, Hence, $x = \frac{-7}{5}$ is a zero (or root) of p(x).

2.3 FACTORIZATION

Factorization is expressing a given polynomial as a product of two or more polynomials.

Ex.1: $x^3 - 15x^2 = x^2(x - 15)$

$\Rightarrow$ x^2 and $x - 15$ are the factors of $x^3 - 15x^2$.

1. **Factorization of polynomials of the form $x^2 - y^2$.**

$x^2 - y^2 = (x + y)(x - y)$

$\Rightarrow$ x + y and x – y are the factors of $x^2 - y^2$.

EX.2: Factorize $81x^2 - 225y^2$

Sol. Let a = 9x and b = 15y [$\because a^2 - b^2 = (a - b)(a + b)$]

$81x^2 - 225y^2 = (9x)^2 - (15y)^2 = (9x + 15y)(9x - 15y)$

$\therefore$ 9x + 15y and 9x – 15y are the factors of $81x^2 - 225y^2$.

2. **Factorization of polynomials by grouping of terms. In this method we group the terms of the polynomials in such a way that we get a common factor out of them.**

EX.3: Factorize $4x^3 - 10y^3 - 8x^2y + 5xy^2$

Sol. $4x^3 - 8x^2y + 5xy^2 - 10y^3 = 4x^2(x - 2y) + 5y^2(x - 2y) = (4x^2 + 5y^2)(x - 2y)$.

3. **Factorization of a trinomial that is a perfect square. A trinomial of the form $x^2 \pm 2xy + y^2$ is equivalent to $(x \pm y)^2$. This identity can be used to factorize perfect square trinomials.**

EX.4: Factorize $16x^2 + \frac{1}{16x^2} - 2$

Sol. $16x^2 + \frac{1}{16x^2} - 2 = (4x)^2 + \left(\frac{1}{4x}\right)^2 - 2(4x)\left(\frac{1}{4x}\right) = (4x)^2 - 2(4x)\left(\frac{1}{4x}\right) + \left(\frac{1}{4x}\right)^2 = \left(4x - \frac{1}{4x}\right)^2$

$\therefore 16x^2 + \frac{1}{16x^2} - 2 = \left(4x - \frac{1}{4x}\right)^2$.

4. **Factorization of a polynomial of the form**

$x^2 + (a+b)x + ab$.

As we have already seen,

$(x+a)(x+b) = x^2 + (a+b)x + ab$

$\therefore$ $x^2 + (a + b)x + ab$ can be factorized as $(x + a)(x + b)$.

EX.5: Factorize $x^2 + 25x + 144$

Sol. Here, the constant term is $144 = (16 \times 9)$ and the coefficient of x is $25 = (16 + 9)$

$\therefore$ $x^2 + 25x + 144$

$= x^2 + 16x + 9x + 144 = x(x + 16) + 9(x + 16) = (x + 16)(x + 9)$.

EX.6: Factorize $x^2 - 8x + 15$

Sol. Here, the constant term is $15 = (-5)(-3)$ and the coefficient of x is $-8 = -5 - 3$.

$\Rightarrow x^2 - 8x + 15 = x^2 - 5x - 3x + 15 = x(x-5) - 3(x-5) = (x-3)(x-5)$

$\therefore x^2 - 8x + 15 = (x-3)(x-5)$.

5. **Factorization of polynomials of the form $ax^2 + bx + c$.**

- Take the product of the constant term and the coefficient of x^2, i.e., ac.
- Now this product ac is to split into two factors m and n such that m + n is equal to the coefficient of x, i.e., b.
- Then we pair one of them, say mx, with ax^2 and the other nx, with c and factorize.

EX.7: $6x^2 + 19x + 15 = (2x + 3)(3x + 5)$.

Sol. Here, $6 \times 15 = 90 = 10 \times 9$ and $10 + 9 = 19$

$\therefore$ $6x^2 + 19x + 15 = 6x^2 + 10x + 9x + 15 = 2x(3x + 5) + 3(3x + 5) = (2x + 3)(3x + 5)$.

6. **Factorization of expressions of the form $x^3 + y^3$ (or) $x^3 - y^3$.**

EX.8: Factorize $27a^3 + 125x^3$

Sol. $27a^3 + 125x^3 = (3a)^3 + (5x)^3 = (3a + 5x)\{(3a)^2 + (5x)^2 - (3a)(5x)\} = (3a + 5x)(9a^2 + 25x^2 - 15ax)$.

EX.9: Factorize $216x^3 - 64y^3$

Sol. $216x^3 - 64y^3 = (6x)^3 - (4y)^3 = (6x - 4y)\{(6x)^2 + (4y)^2 + (6x)(4y)\} = (6x - 4y)(36x^2 + 16y^2 + 24xy)$.

7. **Factorization of expressions of the form**

$x^3 + y^3 + z^3$ when $x + y + z = 0$.

(Given $x + y + z = 0$)

As $x + y + z = 0$, $z = -(x + y)$

$x^3 + y^3 + z^3 = x^3 + y^3 + \{-(x+y)\}^3 = x^3 + y^3 - (x+y)^3 = x^3 + y^3 - \{x^3 + y^3 + 3xy(x+y)\}$

$= -3xy(-z)\ \{\text{Since } x + y = -z\} = 3xyz$

$\therefore$ If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$.

2.4 HCF OF GIVEN POLYNOMIALS

For two given polynomials, f(x) and g(x), r(x) can be taken as the highest common factor, if

1. r(x) is a common factor of f(x) and g(x) and
2. every common factor of f(x) and g(x) is also a factor of r(x).
   Highest common factor is generally referred to a HCF.

EX.10: Find the HCF of $48x^5y^2$ and $112x^3y$.

Sol. Let $f(x) = 48x^5y^2$ and $g(x) = 112x^3y$

Writing f(x) and g(x) as a product of powers of irreducible factors.

$f(x) = 2^4 \cdot 3 \cdot x^5 \cdot y^2$

$g(x) = 2^4 \cdot 7 \cdot x^3 \cdot y$

The common factors with the least exponent are 2^4, x^3 and y

$\therefore$ HCF = $16x^3y$.

EX.11: Find the HCF of $51x^2(x + 3)^3 (x – 2)^2$ and $34x(x – 1)^5 (x – 2)^3$.

Sol. Let $f(x) = 51x^2(x + 3)^3 (x – 2)^2$ and $g(x) = 34x(x – 1)^5 (x – 2)^3$

Writing f(x) and g(x) as the product of the powers of irreducible factors.

$g(x) = 2^4 \cdot 7 \cdot x^3 \cdot y$

$g(x) = 17 \cdot 2 \cdot x(x-1)^5 \cdot (x-2)^3$

The common factors with the least exponents are 17, x and $(x – 2)^2$

$\therefore$ The HCF of the given polynomials = $17 \cdot x \cdot (x-2)^2 = 17x(x-2)^2$.

2.5 LCM OF THE GIVEN POLYNOMIALS

Least Common Multiple or the Lowest Common Multiple is the product of all the factors (taken once) of the polynomials given with their highest exponents respectively.

EX.12: Find the LCM of $18x^3y^2$ and $45x^5y^2z^3$.

Sol. Let $f(x) = 18x^3y^2$ and

$g(x) = 45x^5y^2z^3$

Writing f(x) and g(x) as the product of the powers of irreducible factors.

$f(x) = 2 \cdot 3^2 \cdot x^3 \cdot y^2$

$g(x) = 3^2 \cdot 5 \cdot x^5 \cdot y^2 \cdot z^3$

Now all the factors (taken only once) with the highest exponents are 2, 3^2, 5, x^5, y^2 and z^3.

$\therefore$ The LCM of the given polynomials $= 2 \cdot 3^2 \cdot 5 \cdot x^5 \cdot y^2 \cdot z^3 = 90x^5y^2z^3$.

EX.13: Find the LCM of $51x^2(x + 3)^3 (x – 2)^2$ and $34x(x – 1)^5 (x – 2)^3$

Sol. Writing f(x) and g(x) as the product of powers of irreducible factors.

$f(x) = 17 \cdot 3x^2(x+3)^3 \cdot (x-2)^2$, $g(x) = 17 \cdot x(x-1)^5 \cdot (x-2)^3$

Now all the factors (taken only once) with the highest exponents are 2, 3, 17, $x^2(x – 1)^5$, $(x – 2)^3$ and $(x + 3)^3$.

$\therefore$ The LCM of the given polynomials

$= 2 \cdot 3 \cdot 17 \cdot x^2(x-1)^5 \cdot (x-2)^3 \cdot (x+3)^3 = 102x^2(x-2)^3(x-1)^5(x+3)^3$.

2.6 RELATION BETWEEN THE HCF, THE LCM AND THE PRODUCT OF POLYNOMIALS

If f(x) and g(x) are two polynomials then we have the relation.

(HCF of f(x) and g(x)) × (LCM of f(x) and g(x)) = ± (f(x) × g(x)).

2.6.1 Concept of Square Roots

If x is any variable, then x^2 is called the square of the variable and for x^2, x is called the square root.

Square Root of Monomials

The square root of a monomial can be directly calculated by finding the square roots of the numerical coefficient and that of the literal coefficients and then multiplying them.

EX.14: Find the square root of $1296b^4$.

Sol. Now the given monomial is $1296b^4$.

Square root of $1296b^4 = \sqrt{1296b^4} = \sqrt{1296} \times \sqrt{b^4} = \sqrt{(36)^2} \times \sqrt{(b^2)^2} = 36 \times b^2 \Rightarrow \sqrt{1296b^4} = 36b^2$.

2.6.2 We have four methods to find the square root of an algebraic expression which is not a monomial. They are

1. Method of inspection (using algebraic identities).
2. Method of factorization
3. Method of division
4. Method of undetermined coefficients

- **Method of Inspection**

EX.15: Find the square root of $x^2 + 12xy + 36y^2$.

Sol. $x^2 + 12xy + 36y^2 = (x)^2 + 2(x)(6y) + (6y)^2$

We know that $a^2 + 2ab + b^2 = (a + b)^2$

Now, $\sqrt{x^2 + 12xy + 36y^2} = \sqrt{(x)^2 + 2(x)(6y) + (6y)^2} = \sqrt{(x+6y)^2} = (x+6y)$

$\therefore \Rightarrow \sqrt{(x+6y)^2} = (x+6y)$

EX.16: Find the square root of $a^2x^2 - 2ayx^2 + x^2y^2$.

Sol. $a^2x^2 - 2ayx^2 + x^2y^2 = x^2(a^2 - 2ay + y^2)$

We know that $a^2 - 2ab + b^2 = (a-b)^2$

Now, $\sqrt{a^2x^2 - 2ayx^2 + x^2y^2} = \sqrt{x^2(a^2 - 2ay + y^2)} = \sqrt{x^2(a-y)^2} = \sqrt{[x(a-y)]^2} = x(a-y)$

- **Method of Factorization**

EX.17: Find the square root of $(x^2 - 8x + 15)(2x^2 - 11x + 5)(2x^2 - 7x + 3)$.

Sol. Factorize each expression in the given product, i.e.,

$x^2 - 8x + 15 = x^2 - 5x - 3x + 15 = x(x-5) - 3(x-5) = (x-5)(x-3)$

$2x^2 - 11x + 5 = 2x^2 - 10x - x + 5 = 2x(x-5) - 1(x-5) = (2x-1)(x-5)$

$\sqrt{\left(x^2 - 8x + 15\right)\left(2x^2 - 11x + 5\right)\cdot\left(2x^2 - 7x + 3\right)}$

$= \sqrt{(x-3)^2 (x-5)^2 (2x-1)^2} = (x - 3)(x - 5)(2x - 1)$

$2x^2 - 7x + 3 = 2x^2 - 6x - x + 3 = 2x(x-3) - 1(x-3) = (x-3)(2x-1)$

Hence, the square root of the given expression is (x – 3) (x – 5) (2x – 1).

- **Method of Division**

EX.18: Find the square root of $x^2 - 18x + 81$.

Sol.

$$\begin{array}{r|l}
 & x - 9 \\
\hline
x & x^2 - 18x + 81 \\
 & x^2 \\
\hline
2x - 9 & -18x + 81 \\
 & -18x + 81 \\
\hline
 & \quad\quad 0
\end{array}$$

$\therefore \sqrt{x^2 - 18x + 81} = x - 9$.

- **Method of Undetermined Coefficients**

EX.19: Find the square root of $x^4 + 4x^3 + 10x^2 + 12x + 9$.

Sol. Let us assume the square root to be $ax^2 + bx + c$

$\Rightarrow x^4 + 4x^3 + 10x^2 + 12x + 9 = (ax^2 + bx + c)^2$

We know that $(p + q + r)^2 = p^2 + q^2 + r^2 + 2pq + 2qr + 2rp$

Here, $p = ax^2$, $q = bx$, $r = c$

$\Rightarrow x^4 + 4x^3 + 10x^2 + 12x + 9$

$= (ax^2)^2 + (bx)^2 + c^2 + 2(ax^2)(bx) + 2(bx)(c) + 2(c)(ax^2)$

$\Rightarrow x^4 + 4x^3 + 10x^2 + 12x + 9 = a^2x^4 + b^2x^2 + 2abx^3 + 2cax^2 + 2bcx + c^2$

Now equating the like terms on either sides of the equality sign, we have

$x^4 = a^2x^4 \Rightarrow a^2 = 1 \Rightarrow a = 1$

$4x^3 = 2abx^3 \Rightarrow 2ab = 4 \Rightarrow ab = 2$, but $a = 1$ $b = 2$

$b^2 + 2ca = 10 \Rightarrow 2^2 + 2c = 10 \Rightarrow 2c = 6 \quad \Rightarrow c = 3$

$\therefore$ The square root of the given expression is $ax^2 + bx + c$, i.e., $x^2 + 2x + 3$.

EX.20: Find the square root of $4x^4 – 4x^3 + 5x^2 – 2x + 1$.

Sol. The degree of the given expression is 4, its square root will hence be an expression in degree 2.

Let $\sqrt{4x^2 - 4x^3 + 5x^2 - 2x + 1} = ax^2 + bx + c$

$\Rightarrow (4x^4 - 4x^3 + 5x^2 - 2x + 1) = (ax^2 + bx + c)^2$

$\Rightarrow 4x^4 - 4x^3 + 5x^2 - 2x + 1 = (ax^2)^2 + (bx)^2 + c^2 + 2(ax^2)(bx) + 2(bx)(c) + 2(c)(ax^2)$

$\Rightarrow 4x^4 - 4x^3 + 5x^2 - 2x + 1 = a^2x^4 + 2abx^3 + (b^2 + 2ac)x^2 + 2bcx + c^2$

Now equating the like terms on either sides of the equation, we have

$4x^4 = a^2x^4 \Rightarrow a^2 = 4 \Rightarrow a = 2$

$c^2 = 1 \quad \Rightarrow \quad c = 1$

$2bcx = –2x \qquad 2bc = –2bc = –1,$

$\Rightarrow b = \frac{-1}{c} \Rightarrow \quad b = -1 (\therefore c = 1)$ $b = –1$ ($\because$ = 1)

$\therefore$ The square root of the given expression is $ax^2 + bx + c$, i.e., $2x^2 – x + 1$.

2.7 RATIONAL INTEGRAL FUNCTIONS OF X

A polynomial in x, the exponents in powers of x are non-negative integers and the coefficients of the various powers of x are integers. **Ex.** $11x^2 - 8x + 3, 4x^2 - 5x + 1, 8x^5 - 7x^3 + 8x^2 + 4x + 5$, etc.

2.7.1 Remainder Theorem

"Let P(x) be any polynomial of degree greater then or equal to one. and let 'a' be any real Number. If P (x) is divided by linear Polynomial x-a, then remainder is P(a) "

EX.21: Find the remainder when $x^2 – 8x + 6$ is divided by $2x – 1$.

Sol. Let $q(x) = x^2 – 8x + 6$

$\therefore$ Remainder $= q\left(\frac{1}{2}\right)$ $q\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - 8\left(\frac{1}{2}\right) + 6 = \frac{1}{4} - 4 + 6 = \frac{1}{4} + 2$

$\therefore \quad q\left(\frac{1}{2}\right) = \frac{9}{4}.$

2.7.2 Factor Theorem

If q(x) be a polynomial of degree $n \geq 1$ and if $q(\alpha) = 0$, then $x - \alpha$ is the factor q(x).

EX.22: Is x – 2 a factor of $x^3 + x^2 - 4x - 4$?

Sol. Let $q(x) = x^3 + x^2 - 4x - 4$

$q(2) = 8 + 4 - 8 - 4 = 0$

$\therefore$ x – 2 is a factor of q(x).

EX.23: Find the value of m, if x + 2 is a factor of $x^3 - 4x^2 + 3x - 5m$.

Sol. Let $q(x) = x^3 - 4x^2 + 3x - 5m$.

Given x + 2 is a factor of q(x).

$\therefore q(-2) = 0$

$\Rightarrow (-2)^3 - 4(-2)^2 + 3(-2) - 5m = 0$

$-8 - 16 - 6 - 5m = 0$

$-5m = 30$

$m = -6$.

EX.24: Factorize $x^3 - 2x^2 - 5x + 6$

Sol. Let $f(x) = x^3 - 2x^2 - 5x + 6$

Here, $f(1) = (1)^3 - 2(1)^2 - 5(1) + 6 = 1 - 2 - 5 + 6$

$f(1) = 0 \quad \therefore \quad x - 1$ is a factor of f(x).

To find the other two factors, we use synthetic division.

	1	–2	–5	6
x = 1	0	1	–1	–6
	1	–1	–6	0

$\therefore$ The other two factors are $x^2 - x - 6$ i.e., (x – 3) (x + 2).

$\therefore$ The factorization of the given expression is (x – 1) (x – 3) (x + 2).

2.7.3 Homogeneous Expression

An algebraic expression in which, the degree of all the terms is equal is a homogeneous expression.

Ex. bx + ay is a first-degree homogeneous expression.

$ax^2 + bxy + cy^2$ is a second-degree homogeneous expression.

2.7.4 Symmetric Expressions

f(x, y) is an expression in variables x and y.

If f(x, y) = f(y, x), then f(x, y), is called a symmetric expression.

i.e., If an expression remains same after interchanging the variables x and y is said to be a symmetric expression.

Ex. Consider the expression ax + ay + b

Let f(x, y) = ax + ay + b

$f(y, x) = ay + ax + b = ax + ay + b \Rightarrow f(y, x) = f(x, y)$

$\therefore$ ax + ay + b is symmetric.

2.7.5 Cyclic Expressions

f(x, y, z) is an expression in variable x, y and z.

If f(x, y, z) = f(y, z, x), then f(x, y, z) is cyclic.

Ex. $a^2(a - b) + b^2(b - c) + c^2(c - a)$

Let $f(a, b, c) = a^2(a - b) + b^2(b - c) + c^2(c - a)$

Now, $f(b, c, a) = b^2(b - c) + c^2(c - a) + a^2(a - b) = a^2(a - b) + b^2(b - c) + c^2(c - a)$

$f(b, c, a) = f(a, b, c) \qquad \therefore$ f is cyclic.

EX.25: Factorize $a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)$.

Sol. Let $f(a) = a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)$

$f(b) = b(b^2 - c^2) + b(c^2 - b^2) + c(b^2 - b^2) = b(b^2 - c^2) + b(c^2 - b^2)$

$f(b) = 0$ $\therefore$ By remainder theorem, $(a - b)$ is a factor of the given expression.

The given expression is cyclic, so the other two factors will also be cyclic.

$\therefore$ The other two factors are $(b - c)$ and $(c - a)$.

The given expression may have a constant factor which is non-zero. Let it be m.

$\therefore$ $a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2) = m(a - b)(b - c)(c - a)$

Put $a = 0$, $b = 1$, $c = -1$ in the above equation.

i.e., $0(1^2 - (-1)^2) + 1((-1)^2 - 0) + (-1)(0 - 1^2) = m(0 - 1)(1-(1-1))(-1 - 0)$

$\Rightarrow$ $0 + 1 + 1 = m(-1)(2)(-1)$ $\Rightarrow$ $m = 1$

$\therefore$ the factorization of the given cyclic expressions is $(a - b)(b - c)(c - a)$.

EX.26: Factorize $a(b^3 - c^3) + b(c^3 - a^3) + c(a^3 - b^3)$

Sol. Let $f(a) = a(b^3 - c^3) + b(c^3 - a^3) + c(a^3 - b^3)$

$f(b) = b(b^3 - c^3) + b(c^3 - b^3) + c(b^3 - b^3)$

$b(b^3 - c^3) - b(c^3 - b^3) + 0$

$f(b) = 0$

$\therefore$ By remainder theorem, $a - b$ is the factor of the given expression.

The given expression is cyclic, so the other two factors are also cyclic.

$\therefore$ The factors are $(a - b)(b - c)(c - a)$.

The given expression is of degree 4 but the degree of factor is 3, hence a first-degree cyclic expression is another factor.

Let $m(a + b + c)$ be the factor ($m \neq 0$).

$\therefore$ $a(b^3 - c^3) + b(c^3 - a^3) + c(a^3 - b^3) = m(a + b + c)(a - b)(b - c)(c - a)$

Put $a = 0$, $b = 1$ and $c = 2$ in the above equation.

$0(1^3 - (2)^3) + 1((2)^3 - 0) + 2(0 - 1^3) = m(0 + 1 + 2)(0 - 1)(1 - 2)(2 - 0)$

$\Rightarrow$ $0 + 1(8) + 2(-1) = m(3)(-1)(-1)(2)$

$6 = 6m$ $\Rightarrow$ $m = 1$

$\therefore$ The factorization of the given cyclic expression is $(a - b)(b - c)(c - a)(a + b + c)$.

SUBJECTIVE QUESTIONS

Q.1 Which of the following expressions are polynomial?

(i) $11x + 1$ (ii) $7x^2 - 5x + \sqrt{5}$

(iii) $t^3 - 2t + 1$ (iv) $x^2 - \frac{1}{x^2}$

(v) $\sqrt{y} + 5y - 1$ (vi) $z^{11} - 5z^7 + \frac{1}{4}$

Q.2 Write the coefficient of x^3 in each of the following:

(i) $3x^3 - 3x + 2$ (ii) $14x^3 - 2x^3 + 5x - 7x^2$

(iii) $\sqrt{2}x^2 + 1$ (iv) $\frac{3}{4}x^3 + 2x - 3b$

Q.3 Write the degree of each of the following polynomials:

(i) $3x^2 - 4x + 2$ (ii) $7x^3 + 2x^2 + x$

(iii) $5 - x^2$ (iv) $1 + 2x + 3x^2 - 11x^4$

Q.4 Classify the following as linear, quadratic and cubic polynomials :

(i) $x^3 - 4$ (ii) $x^2 + 1$

(iii) $5x^2 - 3x + \sqrt{7}$ (iv) $1 + 5x$

(v) $4r^3$

Q.5 Find the value of the following polynomial at the indicated value of variables :

(i) $p(x) = 5x^2 - 3x + 7$; at $x = 1$

(ii) $q(y) = 3y^2 - 4y + \sqrt{11}$; at $y = 2$

(iii) $p(t) = 4t^4 + 5t^3 - t^2 + 6$; at $t = a$

Q.6 Find the zeroes of each of the following polynomials:

(i) $p(x) = x - 4$

(ii) $g(x) = 2x + 1$

(iii) $p(x) = (x + 1)(x + 2)$

(iv) $p(x) = (x - 1)(x - 2)(x - 3)$

(v) $p(x) = 7x^2$

(vi) $p(x) = rx + s, r \neq 0$

Q.7 Verify whether the following are zeroes of the polynomial indicated against them :

(i) $p(x) = 5x - 1$, at $x = \frac{1}{5}$

(ii) $p(x) = (x - 2)(x - 5)$, at $x = 2, 5$

(iii) $s(x) = x^2$, at $x = 0, 1$

(iv) $p(x) = 3x^2 - 1$, at $x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(v) $g(x) = 5x^2 + 7x$, at $x = 0, -\frac{7}{5}$

Q.8 Show that -1 is a zero of the polynomial $2x^3 - x^2 + x + 4$.

Q.9 Show that 1 is not a zero of the polynomial $4x^4 - 3x^3 + 2x^2 - 5x + 1$

Q.10 Use remainder theorem to find remainder when p(x) is divided by q(x) in the following questions :

(i) $p(x) = 2x^2 - 5x + 7, q(x) = x - 1$

(ii) $p(x) = x^9 - 5x^4 + 1, q(x) = x + 1$

(iii) $p(x) = 4x^3 - 12x^2 + 11x - 5, q(x) = x - \frac{1}{2}$

(iv) $p(x) = x^4 + x^3 + x^2 - 5x + 1, q(x) = x + 1$

Q.11 Use factor theorem to verify in each of the following that q(x) is a factor of p(x).

(i) $p(x) = 3x^2 - 5x + 2, q(x) = 3x - 2$

(ii) $p(x) = x^4 - x^3 + x - 1, q(x) = x + 1$

(iii) $p(x) = x^5 - x^4 - 4x^2 - 2x + 4, q(x) = x - 2$

Q.12 Find the value of k if $(x - 2)$ is a factor of $2x^3 - 6x^2 + 5x + k$.

Q.13 For what value of m is $2x^3 + mx^2 + 11x + m + 3$ exactly divisible by $(2x - 1)$?

Q.14 Using factor theorem, show that $a - b$ is a factor of $a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)$.

Q.15 Factorize each of the following expressions :

(i) $1 + 2x + x^2$

(ii) $x^2 - 9ax + 18a^2$

(iii) $2xy - x + z - 2zy$

(iv) $p^4 - 81q^4$

(v) $7\sqrt{2}x^2 - 10x - 4\sqrt{2}$

(vi) $x^4 - (2y - 3z)^2$

(vii) $24\sqrt{3}x^3 - 125y^3$

(viii) $a^6x^4 - a^4x^6$

(ix) $125(x - y)^3 + (5y - 3z)^3 + (3z - 5x)^3$

(x) $(x - y)^3 + (y - z)^3 + (z - x)^3$

Q.16 If one of the factors of $x^2 + x - 20$ is $(x + 5)$, find other factor.

Q.17 Simplify :

(i) $\sqrt{2a^2 + 2\sqrt{6}ab + 3b^2}$

(ii) $\sqrt{2}x^2 + 3x + \sqrt{2}$

(iii) $\sqrt{36x^2 + 60x + 25}$

Q.18 Evaluate the following (using identities) :

(i) 103×105 (ii) 98×99

(iii) 104×95 (iv) $(101)^3$

(v) $(399)^3$

Q.19 Write in expanded form :

(i) $\left(\frac{3}{2}x+1\right)^3$ (ii) $\left(x-\frac{2}{3}y\right)^3$

Q.20 Write the expansion of the following :

(i) $(9x + 2y + z)^2$ (ii) $(3x - 2y - z)^2$

Q.21 Find the product of following :

(i) $(x + 3)(x^2 - 3x + 9)$

(ii) $(5a - 3b)(25a^2 + 15ab + 9b^2)$

Q.22 If $x^4+\frac{1}{x^4}=47$. Find the value of $x^3+\frac{1}{x^3}$

Q.23 If $x^4+\frac{1}{x^4}=194$, find $x^3+\frac{1}{x^3}$, $x^2+\frac{1}{x^2}$, and $x+\frac{1}{x}$

Q.24 Factorize :

(i) $27a^2+\frac{1}{64b^3}+\frac{27a^2}{4b}+\frac{9a}{16b^2}$

(ii) $\frac{64}{125}x^3-8-\frac{96}{25}x^2+\frac{48}{5}x$

(iii) $a^3 + 3a^2b + 3ab^2 + b^3 - 8$

Q.25 Find the values of a and b so that $(x + 1)$ and $(x - 1)$ are factors of $x^4 + ax^3 - 3x^2 + 2x + b$.

Q.26 What must be subtracted from $x^3 - 6x^2 - 15x + 80$ so that the result is exactly divisible by $x^2 + x - 12$?

Q.27 Factorize $9z^3 - 27z^2 - 100z + 300$, if its is given that $(3z + 10)$ is a factor of it.

Long Answer Type Questions

Q.28 Let A and B are the remainders when the polynomial $y^3 + 2y^2 - 5ay - 7$ and $y^3 + ay^2 - 12y + 6$ are divided by $y + 1$ and $y - 2$ respectively. If $2A + B = 6$, find the value of a.

Q.29 Without actual division, prove that $a^4 + 2a^3 - 2a^2 + 2a - 3$ is exactly divisible by $a^2 + 2a - 3$.

Q.30 If $(x + 1)$ and $(x - 1)$ are the factors of $mx^3 + x^2 - 2x + n$, find the value of m and n.

Q.31 If $x^2 - 1$ is a factor of $ax^4 + bx^3 + cx^2 + dx + e$, show that $a + c + e = b + d = 0$

Q.32 Find the value of $a^3 - 27b^3$ if $a - 3b = -6$ and $ab = -10$

Q.33 If $x + y + z = 8$ and $xy + yz + zx = 20$, find the value of $x^3 + y^3 + z^3 - 3xyz$.

Q.34 Simplify :

(i) $(a + b)^3 + (a - b)^3 + 6a(a^2 - b^2)$

(ii) $(2a + b + c)^2 + (2a - b - c)^2$

Q.35 Find the value of :

(i) $x^3 + y^3 - 12xy + 64$ when $x + y = -4$

(ii) $x^3 - 8y^3 - 36xy - 216$ when $x = 2y + 6$

(iii) $(x - a)^3 + (x - b)^3 + (x - c)^3 - 3(x - a)(x - b)(x - c)$ when $a + b + c = 3x$

(iv) $(25)^3 - (29)^3 + (4)^3$

Q.36 Prove that $a^3 + b^3 + c^3 - 3abc$

$= \frac{1}{2}(a+b+c)\,[(a - b)^2 + (b - c)^2 + (c - a)^2]$

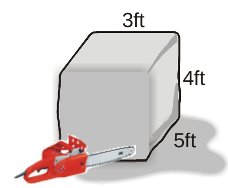
Q.37 If $(3x - 1)^4 = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$, then find the value of $a_4 + 3a_3 + 9a_2 + 27a_1 + 81a_0$.

Q.38 If k and 2k are zeros of $f(x) = x^3 + 4x^2 + 9kx - 90$, find k and all three zeros of f(x).

Q.39 Prove that $(a + b)^3 + (b + c)^3 + (c + a)^3 - 3(a + b)(b + c)(c + a) = 2(a^3 + b^3 + c^3 - 3abc)$

Q.40 The perimeter of a right triangle is 24 centimeters. Three times the length of the longer leg minus two times the length of the shorter leg exceeds the hypotenuse by 2 centimeters. What are the lengths of all three sides?

Q.41 Anshuman is preparing to make an ice sculpture. He has a block of ice that he wants to reduce in size by shaving of the same amount from the length, width, and height. He wants to reduce the volume of the ice block to 24 cubic feet.



(a) Write a polynomial equation to model this situation.

(b) How much should he take from each dimension?

ANALYTICAL QUESTIONS

BASIC LEVEL

Q.1 Which one of the following algebraic expressions is a polynomial in variable x?

(1) $x^2+\frac{2}{x^2}$ (2) $\sqrt{x}+\frac{1}{\sqrt{x}}$

(3) $x^2+\frac{3x^{3/2}}{\sqrt{x}}$ (4) None of these

Q.2 Degree of the polynomial p(x)= $3x^4 + 6x + 7$ is

(1) 4 (2) 5 (3) 3 (4) 1

Q.3 Degree of the polynomial p(x) = (x + 2) (x – 2) is

(1) 2 (2) 1 (3) 0 (4) 3

Q.4 The degree of the polynomial $7x^3y^{10}z^2$ is ________.

(1) 15 (2) 60

(3) 16 (4) None of these

Q.5 $11x^2 – 88x^3 + 14x^4$ is called a ______ polynomial.

(1) quadratic (2) biquadratic

(3) cubic (4) linear

Q.6 Which of the following algebraic expressions is not a polynomial?

(1) $x^2+\frac{3x^{3/2}}{\sqrt{x}}$ (2) $\sqrt{7}x^{33}+3x^{2/3}-8$

(3) 3 (4) 0

Q.7 The degree of a polynomial A is 7 and that of polynomial AB is 56, then find the degree of polynomial B.

(1) 7 (2) 343 (3) 49 (4) 1/49

Q.8 In method of factorization of an algebraic expression, which of the following statements is false?

(1) Taking out a common factor from two or more terms

(2) Taking out a common factor from a group of terms

(3) By using remainder theorem

(4) By using standard identities

Q.9 The LCM and HCF of two monomials is $60x^4y^5a^6b^6$ and $5x^2y^3$ respectively. If one of the two monomials is $15x^4y^3a^6$, then the other monomial is

(1) $12x^2y^3a^6b^6$ (2) $20x^4y^5b^6$

(3) $20x^2y^5b^6$ (4) $15x^2y^5b^6$

Q.10 $\sqrt{a+b-2\sqrt{ab}}$ is _________ where $\sqrt{a}>\sqrt{b}$.

(1) $\sqrt{a}+\sqrt{b}$ (2) $-\sqrt{a}+\sqrt{b}$

(3) $-\sqrt{a}-\sqrt{b}$ (4) $\sqrt{a}-\sqrt{b}$

Q.11 The square root of $y^2+\frac{1}{y^2}+2$ is

(1) $y+\frac{1}{y}$ (2) $y-\frac{1}{y}$

(3) $y^2+\frac{1}{y^2}$ (4) $y^2-\frac{1}{y^2}$

Q.12 If g(x) = $3a^x + 7a^2b – 13ab^2 + 9b^y$ is a homogeneous expression in terms of a and b, then the values of x and y respectively are ________ .

(1) 2, 2 (2) 2, 1 (3) 3, 2 (4) 3, 3

Q.13 The square root of $x^{m^2-n^2}.x^{n^2+2mn}.x^{n^2}$ is

(1) x^{m+n} (2) $x^{(m+n)^2}$

(3) $x^{(m+n)/2}$ (4) $x^{\frac{1}{2}(m+n)^2}$

Q.14 If 3x – 1 is a factor of the polynomial $81x^3 – 45x^2 + 3a – 6$, then a is _______.

(1) $\frac{8}{3}$ (2) $\frac{-7}{3}$ (3) $\frac{-10}{3}$ (4) $\frac{11}{3}$

Q.15 Find the value of a, if (x + 2) is a factor of the polynomial f(x) = $x^3 + 13x^2 + ax + 20$.

(1) – 15 (2) 20 (3) 25 (4) 32

Q.16 One of the factors of $x^3 + 3x^2 – x – 3$ is

(1) x + 1 (2) x + 2 (3) x – 2 (4) x – 3

Q.17 If $3x^3 + 2x^2 – 3x + 4$ = (Ax + B) (x – 1) (x + 2) + C(x – 1) + D for all values of x, then A + B + C + D is

(1) 0 (2) 14

(3) 10 (4) All of these

Q.18 The remainder of $x^4 + x^3 – x^2 + 2x + 3$ when divided by x – 3 is

(1) 105 (2) 108

(3) 10 (4) None of these

Q.19 If x – 3 is a factor of $x^3 + 3x^2 + 3x + p$, then the value of p is
(1) 0 (2) – 63
(3) 10 (4) None of these

Q.20 The remainder obtained when $80x^3 + 55x^2 + 20x + 172$ when divided by x + 2 is ___________ .
(1) –144 (2) 288 (3) –288 (4) 144

Q.21 For the polynomial $p(x) = x^5 + 4x^3 - 5x^2 + x - 1$, one of the factors is
(1) (x + 1) (2) (x – 1)
(3) x (4) (x + 2)

Q.22 If $8x^4 - 8x^2 + 7$ is divided by 2x + 1, the remainder is
(1) $\frac{11}{2}$ (2) $\frac{13}{2}$ (3) $\frac{15}{2}$ (4) $\frac{17}{2}$

Q.23 Factorisation of $a^{2x} - b^{2x}$ is
(1) $(a^x + b^x)(a^x - b^x)$ (2) (x + a) (x + a + c)
(3) (x + b) (x + a + c) (4) (x + b) (x + b + c)

Q.24 If $f\left(\frac{-3}{4}\right) = 0$; then for f(x), which of the following is a factor?
(1) 3x – 4 (2) 4x + 3
(3) –3x + 4 (4) 4x – 3

Q.25 If (x – 3), (x – 3) are factors of $x^3 - 4x^2 - 3x + 18$; then the other factor is
(1) x + 2 (2) x + 3 (3) x – 2 (4) x + 6

Q.26 $f(x) = 3x^5 + 11x^4 + 90x^2 - 19x + 53$ is divided by x + 5 then the remainder is _________ .
(1) 100 (2) – 100 (3) – 102 (4) 102

Q.27 $f(x) = 16x^2 + 51x + 35$ then one of the factors of f(x) is
(1) x – 1 (2) x + 3 (3) x – 3 (4) x + 1

Q.28 If $x^3 - (x + 1)^2 = 2001$ then the value of x is
(1) 14 (2) 13
(3) 10 (4) None of these

Q.29 Factorisation of $a^2 + b^2 + 2(ab + bc + ca)$ is
(1) (a + b) (a + b + 2c) (2) (b + c) (c + a + 2b)
(3) (c + a) (a + b + 2c) (4) (b + a)(b + c + 2a)

Q.30 Factorisation $x^2 + 3\sqrt{2}x + 4$ is
(1) $(x+2\sqrt{2})(x+\sqrt{2})$ (2) $(x+2\sqrt{2})(x-\sqrt{2})$
(3) $(x-2\sqrt{2})(x+\sqrt{2})$ (4) $(x-2\sqrt{2})(x-\sqrt{2})$

Q.31 Factorisation $x^2 - 1 - 2a - a^2$ is
(1) (x – a – 1) (x + a – 1)
(2) (x + a + 1)(x – a – 1)
(3) (x + a + 1) (x – a + 1)
(4) (x – a + 1) (x + a – 1)

Q.32 One of the factor of $(a + 2b)^3 + (2a - c)^3 - (a + 2c)^3 + 3(a + 2b)(2a - c)(a + 2c)$ is
(1) 2a + 2b – 3c (2) 2a – 2b + 3c
(3) 2a + 2b + 3c (4) –2a – 2b – 3c

Q.33 Factors of $\left(x^2 + \frac{x}{6} - \frac{1}{6}\right)$ are
(1) $\frac{1}{6}(2x+1)(3x+1)$ (2) $\frac{1}{6}(2x+1)(3x-1)$
(3) $\frac{1}{6}(2x-1)(3x-1)$ (4) $\frac{1}{6}(2x-1)(3x+1)$

Q.34 Value of $\frac{a^3+b^3+c^3-3abc}{ab+bc+ca-a^2-b^2-c^2}$, when a = – 5, b = – 6, c = 10 is
(1) 1 (2) – 1 (3) 2 (4) – 2

Q.35 The remainder when $x^3 - 3x^2 + 5x - 1$ is divided by x + 1 is ________ .
(1) – 8 (2) – 12 (3) – 10 (4) – 9

Q.36 The expression $x^3 + gx^2 + hx + k$ is divisible by both x and x – 2 but leaves a remainder of 24 when divided by x + 2 then the values of g, h and k are
(1) g = 10, h = –3, k = 0
(2) g = 3, h = – 10, k = 0
(3) g=10, h = –2, k = 3
(4) None of these

Q.37 The expression $Ax^3 + x^2 + Bx + C$ leaves remainder of $\frac{21}{4}$ when divided by 1 – 2x and 18 when divided by x. Given also the expression has a factor of x – 2, the values of A, B and C are -
(1) A = 5, B = –9, C = 3
(2) A = 27, B = – 18, C = 4
(3) A = 4, B = –27, C = 18
(4) None of these

Q.38 If $x^3 + 2x^2 + ax + b$ is exactly divisible by (x + a) and (x – 1), then ________ .
(1) a = – 2 (2) b = – 1 (3) a = – 1 (4) b = 1

Q.39 If $x^3 - hx^2 + kx - 9$ has a factor of $x^2 + 3$, then the values of h and k are
(1) h = 3, k = 3 (2) h = 2, k = 2
(3) h = 2, k = 1 (4) None of these

Q.40 If $h(x) = 2x^3 + (6a^2 - 10)x^2 + (6a + 2)x - 14a - 2$ is exactly divisible by x – 1 but not by x + 1, then the value of a is
(1) 0 (2) –1 (3) 10 (4) 2

Q.41 The remainder when $f(x) = (x^4 - x^3 + 2x - 3)\,g(x)$ is divided by x – 3, given that x – 3 is a factor of g(x) + 3, where g(x) is a polynomial is
(1) 0 (2) –171 (3) 10 (4) 2

Q.42 The value of $ax^2 + bx + c$ when x = 0 is 6. The remainder when dividing by x + 1 is 6. The remainder when dividing by x + 2 is 8. Then the sum of a, b and c is
(1) 8 (2) –1
(3) 10 (4) None of these

Q.43 The remainder when x^{1999} is divided by $x^2 - 1$ is
(1) – x (2) 3x
(3) x (4) None of these

Q.44 For the expression $f(x) = x^3 + ax^2 + bx + c$, if f(1) = f(2) = 0 and f(4) = f(0). The value of a, b & c are
(1) a = – 9, b = 20, c = – 12
(2) a = 9, b = 20, c = 12
(3) a = – 1 b = 2, c = – 3
(4) None of these

Q.45 If x + 1 is a factor of $ax^4 + bx^3 + cx^2 + dx + e = 0$ then _________ .
(1) a + c + e = b + d (2) a + b = c + d
(3) a + b + c + d + e =0 (4) a + c + b = d + e

Q.46 If (x – 3) is the factor of $3x^3 - x^2 + px + q$ then __________ .
(1) p + q = 72 (2) 3p + q = 72
(3) 3p + q = – 72 (4) q – 3p = 72

Q.47 If $ax^3 + 9x^2 + 4x - 1$ is divided by (x + 2), the remainder is –6, then the value of 'a' is
(1) –3 (2) –2 (3) 0 (4) 33/8

Q.48 If $a^3 - 3a^2b + 3ab^2 - b^3$ is divided by (a – b), then the remainder is
(1) $a^2 - ab + b^2$ (2) $a^2 + ab + b^2$
(3) 1 (4) 0

Q.49 The remainder when $f(x) = 3x^4 + 2x^3 - \frac{x^2}{3} - \frac{x}{9} + \frac{2}{27}$ is divided by $g(x) = x + \frac{2}{3}$ is
(1) –1 (2) 1 (3) 0 (4) –2

Q.50 The remainder when $1 + x + x^2 + x^3 + ...+x^{2006}$ is divided by x – 1 is
(1) 2005 (2) 2006 (3) 2007 (4) 2008

Q.51 If (x – 1), (x + 1) and (x – 2) are factors of $x^4 + (p - 3)x^3 - (3p - 5)x^2 + (2p - 9)x + 6$ then the value of p is
(1) 1 (2) 2 (3) 3 (4) 4

Q.52 If the remainder when the polynomial f(x) is divided by x – 1, x + 1 are 6, 8 respectively then the remainder when f(x) is divided by (x – 1) (x+1) is
(1) 7 – x (2) 7 + x (3) 8 – x (4) 8 + x

Q.53 The remainder when x^{100} is divided by $x^2 - 3x + 2$ is
(1) $(2^{100} - 1)x + (-2^{100} + 2)$
(2) $(2^{100} + 1)x + (-2^{100} - 2)$
(3) $(2^{100} - 1)x + (-2^{100} - 2)$
(4) None of these

Q.54 Find the remainder obtained when x^{2007} is divisible by $x^2 - 1$.
(1) x^2 (2) x (3) x + 1 (4) –x

Q.55 The remainder on dividing $121^n - 25^n + 1900^n - (-4)^n$ by 2000 is
(1) 1 (2) 1000 (3) 100 (4) 0

Q.56 The quotient and remainder when $x^{2002} - 2001$ is divided by x^{91} are respectively ________ .
(1) $x^{91\times22}$, 2001 (2) x^{91}, 2001
(3) $x^{91\times21}$, –2001 (4) x^9, –2001

Q.57 Write the degree of the polynomial 0.
(1) 1 (2) 0
(3) N (4) Not defined

Q.58 If the quotient is $3x^2 - 2x + 1$ remainder is 2x – 5 and divisor is x + 2, what is the dividend ?
(1) $3x^3 - 4x^2 + x - 3$ (2) $3x^3 - 4x^2 - x + 3$
(3) $3x^3 + 4x^2 - x + 3$ (4) $3x^3 + 4x^2 - x - 3$

Q.59 If (x – 2) is one factor of $x^2 + ax - 6 = 0$ and $x^2 - 9x + b = 0$, find a + b
(1) 15 (2) 13 (3) 11 (4) 10

Q.60 Given P = Product of x^2y and $\frac{x}{y}$ and Q = Quotient obtained when x^2 is divided by D.
If P = Q, what is the value of D ?
(1) 0 (2) 1 (3) x (4) $\frac{1}{x}$

Q.61 Which of the following is one of the factors of $a^3 + 8b^3 - 64c^3 + 24abc$?
(1) $a + 2b - 4c$ (2) $a - 2b + 4c$
(3) $a + 2b + 4c$ (4) $a - 2b - 4c$

Q.62 Which of the following is a trinomial in p ?
(1) $p^2 + 5$ (2) $p^3 + p^2 + \sqrt{2}$
(3) $\sqrt{p}\left(1+\sqrt{2p}\right)$ (4) $p+\frac{1}{p}+\frac{1}{2}$

Q.63 Identify one of the factors of $x^2+\frac{1}{x^2}+2-2x-\frac{2}{x}$ from the following
(1) $x-\frac{1}{x}$ (2) $x+\frac{1}{x}-1$
(3) $x+\frac{1}{x}$ (4) $x^2+\frac{1}{x^2}$

Q.64 If the factors of $a^2 + b^2 + 2(ab + bc + ca)$ are (a + b + m) and (a + b + nc), find the value of m + n.
(1) 0 (2) 2 (3) 4 (4) 6

Q.65 If $(x^2 + 3x + 5)(x^2 - 3x + 5) = m^2 - n^2$, find m.
(1) $x^2 - 3x$ (2) $3x + 5$
(3) $x^2 + 5$ (4) $x^2 - 5$

Q.66 If $\frac{x}{y}+\frac{y}{x}=-1\left(x,y\neq 0\right)$, what is the value of $x^3 - y^3$?
(1) 1 (2) –1 (3) $\frac{1}{2}$ (4) 0

Q.67 What are the factors of $x^2 + (a + b + c)x + ab + bc$?
(1) (x + a) (x + b + c) (2) (x + a) (x + a + c)
(3) (x + b) (x + a + c) (4) (x + b) (x + b + c)

Q.68 Factorise : $a^4 + 4$
(1) $(a^2 + 2a - 2)(a^2 + 2a + 2)$
(2) $(a^2 - 2a - 2)(a^2 + 2a - 2)$
(3) $(a^2 + 2a - 2)(a^2 - 2a + 2)$
(4) $(a^2 + 2a + 2)(a^2 - 2a + 2)$

Q.69 Identify one of the factors of $x^{12} - y^{12}$
(1) $(x^2 - xy + y^2)$ (2) $(x^4 + x^2y^2 + y^4)$
(3) $(x^2 + xy + y^2)$ (4) $(x^4 - xy + y^4)$

Q.70 If x + y + z = 0, what is the value of $x^3 + y^3 + z^3$?
(1) xyz (2) 2xyz (3) 3xyz (4) 0

Q.71 If a + b + c = 0, evaluate $\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}$
(1) 1 (2) 2 (3) 3 (4) 4

Q.72 Resolve into factors : $6x^3 - 24xy^2 - 3x^2y + 12y^3$
(1) 3(2x – y) (x –2y) (x + y)
(2) 3(2x – y) (x + y) (x + 2y)
(3) 3(2x – y) (x + 2y) (x – 2y)
(4) 3(2x + y) (x – y) (x + 2y)

Q.73 If $x^{\frac{1}{3}}+y^{\frac{1}{3}}+z^{\frac{1}{3}}=0$, which of the following is true ?
(1) $x^3 + y^3 + z^3 = 0$ (2) $x + y + z = 27xyz$
(3) $(x + y + z)^3 = 27xyz$ (4) $x^3 + y^3 + z^3 = 27xyz$

Q.74 Find the value of 'a' if the polynomials $2x^3 + ax^2 + 3x - 5$ and $x^3 + x^2 - 4x - a$ leave the same remainder when divided by x – 1
(1) a = 1 (2) a = – 1
(3) a = –2 (4) a = –2

Q.75 Factorise : $ab(a + b)^2 - 3ab(a + b)$
(1) (a + b) (a – b)ab
(2) (a + b – 5) (2a + b) (a – b)
(3) ab(a + b) (a + b – 3)
(4) ab(2a – b) (2a + b – 6)

Q.76 If x – 1 is a factor of $f(x) = x^3 - 6x^2 + 11x - 6$, which of the following is true ?
(1) f(–x) = 0 (2) f(–1) = 0
(3) f(x) = 0 (4) f(1) = 0

Q.77 If y – 2 and $y - \frac{1}{2}$ are the factors of $py^2 + 5y + r$, which of the following holds good ?
(1) p > r (2) p = r
(3) p < r (4) Both (1) and 3)

Q.78 Which of the following polynomials has –5 as a zero ?
(1) (p – 5) (2) $x^2 - 25$
(3) $p^2 - 5p$ (4) $x^2 + 5$

Q.79 Factorise : $x^2 - z^2 + y^2 - p^2 + 2pz - 2xy$
(1) (x – y – p – z) (x – y – p + z)
(2) (x – y + p – z) (x – y – p + z)
(3) (x + y + p – z) (x + y + p + z)
(4) (x + y – p + z) (x – y – p + z)

Q.80 Find the zeroes of the polynomial $p(z) = (4z + \pi)(z - 4\pi)$

(1) $4\pi, -\frac{\pi}{4}$ (2) $-4\pi, \frac{\pi}{4}$

(3) $4\pi, \frac{\pi}{4}$ (4) $-4\pi, -\frac{\pi}{4}$

Q.81 If (x + 1) is a factor of $x^n + 1$, which of the following statements is true ?

(1) n is an odd integer

(2) n is an even integer

(3) n is a negative integer

(4) n is a positive integer

Q.82 When $x^3 - 2x^2 + ax - b$ is divided by $x^2 - 2x - 3$, the remainder is x – 6. Find the values of a and b.

(1) –2 and –6 (2) 2 and –6

(3) –2 and 6 (4) 2 and 6

Q.83 If $x^2 + x + 1$ is a factor of the polynomial $3x^3 + 8x^2 + 8x + 3 + 5k$, what is the value of k ?

(1) 0 (2) 2/5

(3) 5/2 (4) –1

Q.84 If (x – 1) is a factor of polynomial f(x) but not of g(x), it must be a factor of which of the following polynomials ?

(1) f (x) g (x) (2) –f(x) + g(x)

(3) f(x) – g(x) (4) {f(x) + g(x)}g(x)

Q.85 If (x – a) (x – b) are factors of polynomial g(x), which of the following statements is correct ?

(1) $g(a) = 0, g(b) \neq 0$ (2) $g(a) = 0, g(b) = 0$

(3) $g(a) \neq 0, g(b) \neq 0$ (4) $g(a) \neq 0, g(b) = 0$

Q.86 If $x^{2014} + 2014$ is divided by (x + 1), what is the remainder ?

(1) 2014 (2) 1

(3) 2013 (4) 2015

Q.87 If $x^2 + kx + 6 = (x + 2)(x + 3)$ for all x, what is the value of k ?

(1) 1 (2) 6 (3) 5 (4) 3

Q.88 If $x+\frac{1}{x}=3$, find the value of $x^2+\frac{1}{x^2}$?

(1) 9 (2) 11 (3) 7 (4) 8

Q.89 Find the one of the factors of $(x - 1) - (x^2 - 1)$.

(1) $x^2 - 1$ (2) x + 1

(3) x – 1 (4) x + 4

Q.90 What is the product of

$\left(y-\frac{1}{y}\right)\left(y+\frac{1}{y}\right)\left(y^2+\frac{1}{y^2}\right)$?

(1) $y^4+\frac{1}{y^4}$ (2) $y^4+\frac{1}{y^2}+2$

(3) $y^4-\frac{1}{y^4}$ (4) $y^3+\frac{1}{y^3}-2$

Q.91 If $(x + y)^3 - (x - y)^3 - 6y(x^2 - y^2) = ky^3$, find k

(1) 1 (2) 2 (3) 4 (4) 8

Q.92 Factorise $\frac{a^2}{b^2}+2+\frac{b^2}{a^2}$

(1) $\left(\frac{a}{b}+\frac{b}{a}\right)^2$ (2) $\left(1+\frac{a}{b}\right)^2$

(3) $\left(1-\frac{b}{a}\right)^2$ (4) $\left(\frac{b}{a}-\frac{a}{b}\right)^2$

Q.93 Factorise $4a^2 + 9b^2 + c^2 + 12ab + 4ac + 6bc$.

(1) $(2a + 3b + c)^2$ (2) $(a + 3b + 2c)^2$

(3) $(a - 3b + 2c)^2$ (4) $(2a + 3b - c)^2$

Q.94 What is the remainder obtained when the polynomial p(x) is divided by (b – ax) ?

(1) $p\left(\frac{-b}{a}\right)$ (2) $p\left(\frac{a}{b}\right)$

(3) $p\left(\frac{b}{a}\right)$ (4) $p\left(\frac{-a}{b}\right)$

Q.95 If a + b + c = 10, $a^2 + b^2 + c^2 = 38$ and $a^3 + b^3 + c^3 = 160$, find the value of abc

(1) 45 (2) 15 (3) 10 (4) 30

Q.96 What are the factors of $x^3 + x^2 - \frac{1}{x^2}+\frac{1}{x^3}$

(1) $\left(x^2+1\right)\left(x+\frac{1}{x}-1+\frac{1}{x^2}\right)$

(2) $(x+1)\left(x^2+\frac{1}{x^2}-1+\frac{1}{x}-x\right)$

(3) $\left(x+\frac{1}{x}\right)\left(x^2+x-1-\frac{1}{x}+\frac{1}{x^2}\right)$

(4) $\left(x^2+\frac{1}{x^2}\right)\left(x+\frac{1}{x}-1\right)$

Q.97 Evaluate $\dfrac{(3.78)^2-(2.22)^2}{1.56}$

(1) 6 (2) 3 (3) 9 (4) 15

Q.98 Let R_1 and R_2 be the remainders when the polynomials $f(x) = 4x^3 + 3x^2 - 12ax - 5$ and $g(x) = 2x^3 - ax^2 - 6x + 2$ are divided by $(x - 1)$ and $(x - 2)$ respectively. If $3R_1 + R_2 + 28 = 0$, find the value of 'a'

(1) 0 (2) – 1 (3) 1 (4) 32

Q.99 What is the value of $\dfrac{(2.3)^3-0.027}{(2.3)^2+0.69+0.09}$?

(1) 2 (2) 3 (3) 2.327 (4) 2.273

Q.100 If $(x - 1)^7 = a_7x^7 + a_6x^6+a_5x^5+.... +a_1x +a_0$, what is the value of $a_7 + a_6 + a_5 + ... + a_1 + a_0$?

(1) 0 (2) 1 (3) 128 (4) 64

Q.101 Resolve into factors $1 + a + b + c + ab + bc + ca + abc$

(1) $(1 + a) (1 + b) (1 + c)$

(2) $(a + b + c + 1) (a - b - c)$

(3) $(a + b) (b + c) (c + a) + 1$

(4) $(a^2 + b^2 + c^2) (1 - a - b - c)$

Q.102 Which of the following are the factors of $4x^2 - y^2 + 2x - 2y - 3xy$?

(1) $(x + y)$ and $(4x + y - 2)$

(2) $(x - y)$ and $(4x - y + 2)$

(3) $(x + y)$ and $(4x - y - 2)$

(4) $(x - y)$ and $(4x + y + 2)$

Q.103 Find the value of the polynomial $x^2 - x - 1$ at $x = -1$.

(1) –3 (2) 1 (3) –1 (4) 0

Q.104 Given that $(1 - x) (1 + x + x^2 + x^3 + x^4)$ is $\frac{31}{32}$ and x is a rational number, what is $1 + x + x^2 + x^3 + x^4 + x^5$?

(1) $\frac{31}{64}$ (2) $\frac{63}{32}$ (3) $\frac{63}{64}$ (4) $\frac{31}{32}$

Q.105 If $x^3 + y^3 + z^3 - 3xyz = k (x + y + z) \{(x - y)^2 + (y - z)^2 + (z - x)^2\}$, find k.

(1) 1 (2) $\frac{1}{4}$ (3) $\frac{1}{2}$ (4) $\frac{1}{3}$

ADVANCE LEVEL

Q.1 If $x + \frac{1}{x} = 5$, the value of $\dfrac{x^4+1}{x^2}$ is :

(1) 21 (2) 23 (3) 25 (4) 30

Q.2 Let $x = (2008)^{1004} + (2008)^{-1004}$ and $y = (2008)^{1004} - (2008)^{-1004}$ then the value of $(x^2 - y^2)$ is equal to:

(1) 4 (2) –4 (3) 0 (4) None

Q.3 If $a + b + c = 0$ then value of $\dfrac{a^2}{bc}+\dfrac{b^2}{ca}+\dfrac{c^2}{ab}$ is :

(1) 1 (2) –1 (3) 0 (4) 3

Q.4 If $x + y = -4$, then the value of $x^3 + y^3 - 12xy + 64$ will be

(1) 0 (2) 128 (3) 64 (4) –64

Q.5 F(x) is a polynomial in x. When F(x) is divided by (x -2), the remainder obtained is 3, when the same polynomial is divided by (x –3), then remainder obtained is 2. What is the remainder when F(x) is divided by (x –3) (x–2)

(1) $-x + 5$ (2) $-\frac{5}{3}x + 7$ (3) 0 (4) 5

Q.6 Determine the value of a for which the polynomial $2x^4 - ax^3 + 4x^2 + 2x + 1$ is divisible by $1 - 2x$.

(1) 25 (2) 26 (3) 28 (4) 30

Q.7 $x^{831+} y^{831}$ is always divisible by

(1) $x-y$ (2) $x^2 + y^2$

(3) $x + y$ (4) None of these

Q.8 The remainder when x^{45} is divide by $x^2 - 1$ is

(1) 2x (2) –x (3) 0 (4) x

Q.9 Find the square root of $\dfrac{x^2}{9}+\dfrac{9}{4x^2}-\dfrac{x}{3}-\dfrac{3}{2x}+\dfrac{5}{4}$

(1) $\dfrac{2x}{3}+\dfrac{3}{2x}-\dfrac{1}{2}$ (2) $\dfrac{x}{3}-\dfrac{3}{2x}+1$

(3) $\dfrac{3}{x}+\dfrac{2}{3x}-\dfrac{1}{2}$ (4) $\dfrac{x}{3}+\dfrac{3}{2x}-\dfrac{1}{2}$

Q.10 Find the square root of $\dfrac{a^2}{4}+\dfrac{1}{a^2}-\dfrac{1}{a}+\dfrac{a}{2}-\dfrac{3}{4}$.

(1) $\dfrac{a}{2}-\dfrac{1}{a}+\dfrac{1}{2}$ (2) $\dfrac{a}{2}+\dfrac{2}{a}-1$

(3) $\dfrac{a}{2}+\dfrac{1}{a}-\dfrac{1}{2}$ (4) $\dfrac{a}{2}-\dfrac{2}{a}-\dfrac{1}{2}$

Q.11 If a + b + c = 0, find $\frac{b^2+c^2+a^2}{b^2-ca}$

(1) 1 (2) 2 (3) 6 (4) 8

Q.12 a, b, c are positive integers such that $a^2 + 2b^2 - 2bc = 100$ and $2ab - c^2 = 100$. Then $\frac{a+b}{c}$ is

(1) 10 (2) 100 (3) 2 (4) 1000

Q.13 If a, b, c are integers such that $a^2 + 2b^2 - 2ab = 169$ and $2ab - c^2 = 169$ then a + b + c is

(1) 0 (2) 169 (3) 13 (4) 39

Q.14 If a = 2012, b = –1005, c = –1007, then the value of $\frac{a^4}{b+c}+\frac{b^4}{c+a}+\frac{c^4}{a+b}+3abc$ is

(1) 2012 (2) 1 (3) 0 (4) $(2012)^3$

Q.15 If x > y > 0 and $\frac{x+y}{x-y}=\sqrt{2}$, the value of $\frac{x^2+y^2}{xy}$ is

(1) 5 (2) 4 (3) 1 (4) 6

Q.16 If x, y are positive real numbers satisfying the system of equations $x^2+y\sqrt{xy}$ = 336, $y^2+x\sqrt{xy}$ =112, then x + y equals

(1) $\sqrt{448}$ (2) $\sqrt{224}$ (3) 20 (4) 40

Q.17 If ab + bc + ca = 0, find the value of $\frac{1}{a^2-bc}+\frac{1}{b^2-ca}+\frac{1}{c^2-ab}$.

(1) –1 (2) 2 (3) 1 (4) 0

Q.18 If a + b + c = 0, then find the value of $\frac{(b+c)^2}{3bc}+\frac{(c+a)^2}{3ac}+\frac{(a+b)^2}{3ab}$.

(1) 0 (2) –1 (3) 1 (4) 2

Q.19 The remainder when $f(x) = 4x^3 - 3x^2 + 2x - 1$ is divided by 2x + 1 is ___.

(1) 1 (2) $\frac{-3}{4}$ (3) $\frac{-13}{4}$ (4) $\frac{-7}{4}$

Q.20 Let R_1 and R_2 are the remainders when the polynomials $x^3 + 2x^2 - 5ax - 7$ and $x^3 + ax^2 - 12x + 6$ are divided by x+ 1 and x–2 respectively. If $2R_1 + R_2 = 6$, find the value of a.

(1) 2 (2) –2 (3) –1 (4) 0

Q.21 Find the values of a and b so that the polynomial $x^3 - ax^2 - 13x + b$ has (x –1) and (x+3) as factors.

(1) a = 3 and b=15 (2) a =3, b = –15
(3) a = –3, b =15 (4) b = –13, b = –15

Q.22 What must be added to $x^4 + 2x^3 - 2x^2 + x$, So that the result is exactly divisible by $x^2 + 2x - 3$.

(1) 2x +3 (2) –2x + 3 (3) x –3 (4) –x + 3

Q.23 If $f(x) = x^4 - 2x^4 + 3x^2 - ax + b$ is a polynomial such that when it is divided by x –1 and x + 1, the remainders are 5 and 19 respectively. Determine the remainder when f(x) is divided by x–2.

(1) 10 (2) 5 (3) 8 (4) 2

Q.24 What is the first degree expression to be added to $16x^6 + 8x^4 - 2x^3 + x^2 + 2x + 1$ in order to make it a perfect square?

(1) $\frac{5}{2}x+\frac{15}{16}$ (2) $-\frac{5}{2}x-\frac{15}{16}$
(3) $-\frac{5}{2}x+\frac{15}{16}$ (4) $+\frac{2}{2}x-\frac{15}{16}$

Q.25 The LCM of $x^2 - 16$ and $2x^2 - 9x + 4$ is

(1) (2x + 1) (x + 4) (x – 4)
(2) $(x^2 + 16)$ (2x + 1)
(3) 2(1 – 2x) (x + 4) (x – 4)
(4) (2x – 1) (x + 4) (x – 4)

Q.26 If (x + 1) (x + 2) (x + 3) (x + k) + 1 is a perfect square, then the value of k is

(1) 4 (2) 5 (3) 6 (4) 7

Q.27 If the polynomial $x^{19} + x^{17} + x^{13} + x^{11} + x^7 + x^5 + x^3$ is divided by $(x^2 + 1)$, then the remainder is

(1) 1 (2) $x^2 + 4$ (3) –x (4) x

Q.28 If (x – 2) is a common factor of $x^3 - 4x^2 + ax + b$ and $x^3 - ax^2 + bx + 8$, then the values of a and b are respectively

(1) 3 and 5 (2) 2 and – 4
(3) 4 and 0 (4) 0 and 4

Q.29 The polynomial $11a^2 - 12\sqrt{2}a + 2$ on factorization gives

(1) $(11a+\sqrt{2})(a-\sqrt{2})$
(2) $(a-\sqrt{2})(11a-\sqrt{2})$
(3) $(a+11)(a+\sqrt{2})$
(4) $(11a-\sqrt{2})(a+\sqrt{2})$

Q.30 $ax^4 + bx^3 + cx^2 + dx + e$ is exactly divisible by $x^2 - 1$, when :

(1) a + b + c + e = 0 (2) a + c + e = 0
(3) a + b = 0 (4) a + c + e = b + d = 1

Q.31 If $f(x) = ax^2 + bx + c$ is divided by $(bx + c)$, then the remainder is __________ .

(1) $\frac{c^2}{b^2}$ (2) $\frac{ac^2}{b^2}+2c$

(3) $f\left(-\frac{c}{b}\right)$ (4) $\frac{ac^2+2b^2c}{b^2}$

Q.32 $x^n - y^n$ is divisible by $x + y$, when n is ______ .

(1) An odd positive integer
(2) An even positive integer
(3) An integer
(4) None of these

Q.33 For what values of n, $(x + y)$ is a factor of $(x - y)^n$.

(1) for all values of n
(2) 1
(3) only for odd numbers
(4) none of these

Q.34 The square root of $\frac{x^2}{y^2}+\frac{y^2}{4x^2}-\frac{x}{y}+\frac{y}{2x}-\frac{3}{4}$ is

(1) $\frac{x}{y}-\frac{1}{2}-\frac{y}{2x}$ (2) $\frac{x}{y}+\frac{1}{2}-\frac{y}{2x}$

(3) $\frac{x}{y}+\frac{1}{2}+\frac{y}{2x}$ (4) $\frac{x}{y}-\frac{1}{4}-\frac{y}{2x}$

Q.35 On simplifying $(a + b)^3 + (a - b)^3 + 6a(a^2 - b^2)$ we get

(1) $8a^2$ (2) $8a^2b$ (3) $8a^3b$ (4) $8a^3$

Q.36 If $(x + y + z) = 1$, $xy + yz + zx = -1$, $xyz = -1$, then the value of $x^3 + y^3 + z^3$ is

(1) –1 (2) 1 (3) 2 (4) – 2

Q.37 If the LCM and HCF of two polynomials are $90\ m^5a^6b^3x^2$ and $m^3\ a^5$ respectively and also one of the monomial is $18\ m^5a^6x^2$, then the other monomial is

(1) $5\ m^3a^5b^3$ (2) $15\ m^5a^3b^2$

(3) $5\ m^5a^3b^5$ (4) $15\ m^3a^5b^4$

Q.38 If the expressions $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + a$ on dividing by $x - 4$ leave the same remainder, then the value of a is

(1) 1 (2) 0 (3) 2 (4) –1

Q.39 If the polynomial $x^6 + px^5 + qx^4 - x^2 - x - 3$ is divisible by $x^4 - 1$, then the value of $p^2 + q^2$ is

(1) 1 (2) 5 (3) 10 (4) 13

Q.40 The value of m, if $2x^m + x^3 - 3x^2 - 26$ leaves a remainder of 226 when it is divided by $x - 2$.

(1) 0 (2) 7
(3) 10 (4) All of these

Q.41 Given the polynomial is exactly divided by x + 1, and when it is divided by 3x – 1, the remainder is 4. The polynomial gives a remainder hx + k when divided by $3x^2 + 2x - 1$ then the values of h and k are

(1) h = 2, k = 3 (2) h = 3, k = 3
(3) h = 3, k = 2 (4) None of these

Q.42 The value of k for which (x+2) is a factor of $(x+1)^7 + (3x + k)^3$ is _____.

(1) –7 (2) 7 (3) –1 (4) $-6 - 3^{(7/3)}$

Q.43 The remainder when $x^4 - y^4$ is divided by x –y is _____ is .

(1) 0 (2) x + y (3) $x^2 - y^2$ (4) $2y^4$

Q.44 When $p(x) = x^3 + ax^2 + 2x + a$ is divided by (x + a), the remainder is _____.

(1) 0 (2) a (3) –a (4) 2a

Q.45 $x^{12} - y^{12} =$

(1) $(x-y)(x^2+xy+y^2)(x+y)(x^2-xy+y^2)(x^2+y^2)(x^4-x^2y^2+y^4)$

(2) $(x+y)(x^2-xy+y^2)(x+y)(x^2-xy+y^2)(x^2+y^2)(x^4-x^2y^2+y^4)$

(3) $(x+y)(x^2+xy-y^2)(x+y)(x^2-xy+y^2)(x^2+y^2)(x^4-x^2y^2+y^4)$

(4) $(x-y)(x^2-xy+y^2)(x+y)(x^2-xy+y^2)(x^2+y^2)(x^4-x^2y^2+y^4)$

Q.46 If $x = \frac{a-b}{a+b}$, $y = \frac{b-c}{b+c}$, $z = \frac{c-a}{c+a}$, then the value of $\frac{(1+x)(1+y)(1+z)}{(1-x)(1-y)(1-z)}$ is _____.

(1) abc (2) $a^2b^2c^2$ (3) 1 (4) –1

Q.47 If (x + 2) and (x –1) are factors of $(x^3 + 10x^2 + mx + n)$, then the value of m, n respectively are ___.

(1) –5, 5 (2) 7, 18 (3) 7, –18 (4) –5, –18

Q.48 Given that x = 2 is a solution of $x^3 - 7x + 6 = 0$. The other solutions are _____.

(1) –1,3 (2) 1, –3 (3) 1, –2 (4) –1, –2

Q.49 If (x+ k) is a common factor of $f(x) = (x^2 + px + q)$ and $g(x) = (x^2 + lx + m)$, then the value of k is ______.

(1) $l + p$ (2) $m - q$ (3) $\frac{l-p}{m-q}$ (4) $\frac{m-q}{l-p}$

Q.50 The product $(a + b)(a - b)(a^2 - ab + b^2)(a^2 + ab + b^2)$ is equal to ___.

(1) $a^6 + b^6$ (2) $a^6 - b^6$ (3) $a^3 - b^3$ (4) $a^3 + b^3$

Q.51 The value of $(x-a)^3 + (x-b)^3 + (x-c)^3 - 3(x-a)(x-b)(x-c)$, when $a + b + c = 3x$ is ____.

(1) 3 (2) 2 (3) 1 (4) 0

Q.52 Value of R, if $\frac{a^2 - 19a - 25}{a-7} = a - 12 + \frac{R}{a-7}$ is __.

(1) –109 (2) –88 (3) –84 (4) –64

Q.53 When $(x^3 - 2x^2 + px - q)$ is divided by $(x^2 - 2x - 3)$, the remainder is $(x-6)$. The value of p and q respectively are _____.

(1) – 2, –6 (2) 2, –6 (3) –2, 6 (4) 2, 6

Q.54 Find the remainder when the expression $3x^3 + 8x^2 - 6x + 1$ is divided by $x + 3$.

(1) 1 (2) 10 (3) 6 (4) 0

Q.55 If $x^2 - 1$ is a factor of $ax^4 + bx^3 + cx^2 + dx + e$, then

(1) $a + b + e = c + d$ (2) $a + b + c = d + e$
(3) $b + c + d = a + e$ (4) None of these

Q.56 If a, b, c are all non- zeroes and a + b + c =0, then $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{b^2}{ab} =$ _______.

(1) 0 (2) 1 (3) 2 (4) 3

Q.57 Length, breadth and height of a cuboidal tank are $(x-3y)$m, $(x+3y)$m and (x^2+9y^2)m respectively. Find the volume of the tank.

(1) $(x^3 + 3xy + 27y^3)m^3$
(2) $(x^4 + 2x^2y^2 + 81y^4)\ m^3$
(3) $(x^4 - 81y^4)m^3$
(4) $(x^4 + 81y^4)\ m^3$

Q.58 A rectangular field has an area $(35x^2 + 13x - 12)m^2$. What could be the possible expression for length and breadth of the field ?

(1) $(5x + 4)$m and $(7x - 3)$ m
(2) $(3x + 9)$ m and $(7x - 12)$m
(3) Both (1) and (2)
(4) None of these

Q.59 Santosh has Rs $(x^3 - 3x^2 + 4x + 50)$. He want to by chocolates each of cost Rs $(x - 3)$. After buying maximum number of chocolates with him money, how much money is left with him ?

(1) Rs 50 (2) Rs 40 (3) Rs 62 (4) Rs 20

Q.60 Area of a rectangular field is $(2x^3 - 11x^2 - 4x + 5)$ sq. units and side of a square field is $(2x^2 + 4)$ units. Find the difference between areas (in sq. units).

(1) $4x^4 - 2x^3 - 27x^2 - 4x + 11$
(2) $4x^4 - 2x^3 + 27x^2 + 4x + 11$
(3) $4x^4 + 27x^2 + 4x - 11$
(4) $4x^4 + 2x^3 + 27x^2 + 4x + 11$

Q.61 Vikas has Rs $(x^3 + 2ax + b)$, with this money he can buy exactly $(x-1)$ jeans or $(x+1)$ shirts with no money left. How much money Vikas has, If x =4 ?

(1) Rs 80 (2) Rs 120 (3) Rs 30 (4) Rs 60

KNOWLEDGE BOOSTER

Q.62 Which of the following statements is INCORRECT?

(1) Every non-zero constant polynomial has zero roots.
(2) Zero polynomial has zero root.
(3) Every linear polynomial has exactly one root.
(4) If $x - a$ is the root of $p(x) = 0$, then $p(a) = 0$.

Q.63 If $(5x^2 + 14x + 2)^2 - (4x^2 - 5x + 7)^2$ is divided by $(x^2 + x + 1)$, then quotient 'q' and remainder 'r' respectively, are ____.

(1) $(x^2 + 19x - 5)$, 0
(2) $9(x^2 + 19x - 5)$, 0
(3) $(x^2 + 19x - 5)$, 1
(4) $9(x^2 + 19x - 5)$, 1

Q.64 Select the CORRECT statement

(1) If $x = \frac{\sqrt{3}+1}{\sqrt{3}-1} + \frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}-2}{\sqrt{3}+2}$, then the value of $x^2 + \left(\frac{39}{x}\right)^2$ is 110.
(2) Every integer is a whole number.
(3) Between two rational numbers, there exist infinite number of integers.
(4) Nose of these

PREVIOUS YEAR QUESTIONS

Q.1 H.C.F and L.C.M of expression $(x^3 - 1)$ and A are $(x - 1)$ and $(x^6 - 1)$ respectively. Then the value of A is : **[Raj. NTSE Stage - 1 2005]**

(1) $x^3 + 1$ (2) $x^4 - x^3 + x - 1$
(3) $(x - 1)(x^2 - x + 1)$ (4) $(x - 1)(x^2 + x + 1)$

Q.2 H.C.F of $x^2 + 5x + 6$ and $x^3 + 27$ is : **[Raj. NTSE Stage - 1 2006]**

(1) $x + 2$ (2) $x - 2$ (3) $x - 3$ (4) $x + 3$

Q.3 The value of x in the equation $\frac{x-1}{x+1} = \frac{x+5}{2x+5}$ is : **[Raj. NTSE Stage - 1 2007]**

(1) –1 (2) –5 (3) 1 (4) 5

Q.4 One of the factors of the expression $(2x - 3y)^2 - 7(2x - 3y) - 30$ is : **[Raj. NTSE Stage - 1 2007]**

(1) $2x - 3y - 10$ (2) $2x - 3y + 10$
(3) $3x - 2y + 5$ (4) $6x - 4y - 15$

Q.5 L.C.M of $x^3 + x^2 + x + 1$ and $x^3 - x^2 + x - 1$ is : **[Raj. NTSE Stage - 1 2007]**

(1) $x^4 + 1$ (2) $x^4 - 1$ (3) $x^2 + 1$ (4) $x^2 - 1$

Q.6 If $a^2 + 2b = 7$, $b^2 + 4c = -7$ and $c^2 + 6a = -14$, then the value of $(a^2 + b^2 + c^2)$ is : **[IJSO -2009]**

(1) 14 (2) 25 (3) 36 (4) 47

Q.7 If $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = \frac{10}{3}$ and $x + y = 10$, then the value of xy will be : **[NSTSE 2010]**

(1) 16 (2) 9 (3) 2 (4) 10

Q.8 If $x + \frac{1}{x} = 3$, then the value of $x^6 + \frac{1}{x^6}$ is : **[Raj. NTSE Stage - 1 2013]**

(1) 927 (2) 114 (3) 364 (4) 322

Q.9 If the zero of the polynomial $f(x) = k^2x^2 - 17x + k + 2$ $(k > 0)$ are reciprocal of each other, then the value of k is : **[Delhi NTSE Stage - 1 2013]**

(1) 2 (2) –1 (3) –2 (4) 1

Q.10 If $(a - 5)^2 + (b - c)^2 + (c - d)^2 + (b + c + d - 9)^2 = 0$, then the value of $(a + b + c)(b + c + d)$ is : **[Haryana NTSE Stage - 1 2013]**

(1) 0 (2) 11 (3) 33 (4) 99

Q.11 A cubic polynomial p(x) is such that p(1) = 1, p(2) = 2, p(3) and p(4) = 5, then the value of p(6) is : **[Haryana NTSE Stage - 1 2013]**

(1) 16 (2) 13 (3) 10 (4) 7

Q.12 If $x + y + z = 1$, then $1 - 3x^2 - 3y^2 - 3z^2 + 2x^3 + 2y^3 + 2z^3$ is equal to : **[Haryana NTSE Stage - 1 2013]**

(1) 6xyz (2) 3xyz (3) 2xyz (4) xyz

Q.13 The sum of real values of y satisfying the equations $x^2 + x^2y^2 + x^2y^4 = 525$ and $x + yx + xy^2 = 35$ is : **[Haryana NTSE Stage - 1 2013]**

(1) 15 (2) 10 (3) 5/2 (4) 3/2

Q.14 If a, b, c and d are natural numbers such that $a^5 = b^6$, $c^3 = d^4$, and $d - a = 61$, then the smallest value of $c - b$ is : **[Haryana NTSE Stage - 1 2013]**

(1) 61 (2) 122 (3) 239 (4) 593

Q.15 If x,y,z are positive numbers and a,b,c are rational numbers, then the value of **[Raj. NTSE Stage - 1 2013]**

$$\frac{1}{1+x^{b-c}+x^{a-c}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{b-a}+x^{c-a}}$$

(1) – 1 (2) 0
(3) 1 (4) None of these

Q.16 If $x^2 - x - 1 = 0$, then the value of $x^3 - 2x + 1$ is **[Haryana NTSE Stage - 1 2014]**

(1) 0 (2) 2 (3) $\frac{1+\sqrt{5}}{2}$ (4) $\frac{1-\sqrt{5}}{2}$

Q.17 If x% of y is equal to 1% of z, y% of z is equal to 1% of x and z% of x is equal to 1% of y, then the value of xy + yz + zx is – **[Haryana NTSE Stage - 1 2014]**

(1) 1 (2) 2 (3) 3 (4) 4

Q.18 If $(x + a)^2 + (y + b)^2 = 4(ax + by)$, where x, a, y, b are real the value of xy – ab is : **[West Bengal NTSE Stage - 1 2014]**

(1) a (2) 0
(3) b (4) None of these

Q.19 If (x + 2), is a factor of $2x^3 - 5x + K$, then the value of k is **[Raj. NTSE Stage - 1 2016]**

(1) 6 (2) –6 (3) 26 (4) –26

Q.20 If a + b + c = 0, then the value of **[Raj. NTSE Stage - 1 2016]**

$$\frac{(a+b)^2}{ab}+\frac{(b+c)^2}{bc}+\frac{(c+a)^2}{ca} \text{ is}$$

(1) 1 (2) 2 (3) 3 (4) – 3

Q.21 If the value of quadratic polynomial p(x) is o only at x = -1 and p (-2) = 2 then the value of p(2) is **[NTSE Stage - 2 2016]**

(1) 18 (2) 9 (3) 6 (4) 3

Q.22 The cube root of $x + y + 3x^{1/3} y^{1/3} (x^{1/3} +y^{1/3})$ is **[Raj. \NTSE Stage - 1 2017]**

(1) x + y (2) $x^{1/3} + y^{1/3}$
(3) $(x + y)^{1/3}$ (4) $(x +y)^{1/3}$

Q.23 If $(x + \sqrt{2})$ is a factor of $kx^2 – \sqrt{2}x + 1$, then the value of k is : **[Raj.\ NTSE Stage - 1 2017]**

(1) $-\frac{3}{2}$ (2) $-\frac{2}{3}$ (3) $\frac{2}{3}$ (4) $\frac{2}{3}$

Q.24 If a = x –y, b = y – z and c =z –x then the value of $a^3 + b^3 + c^3$ is ; **[Raj. NTSE Stage - 1 2017]**

(1) 3(x–y) (y– z) (z – x) (2) $(x – y)^3 (y–z)^3 (z–x)^3$
(3) $(x + y + z)^3$ (4) $x^3 + y^3 + z^3$

Q.25 When a polynomial p(x) is divided by x -1, the remainder is 3. When p(x) is divided by x -3, the remainder is 5. If r(x) is the remainder when p(x) is divided by (x – 1)(x–3), then the value of r(–2) is **[NTSE Stage - 1 2016]**

(1) –2 (2) –1 (3) 0 (4) 4

Q.26 The value(s) of k for which $x^2 + 5kx + k^2 + 5$ is exactly divisible by x + 2 but not by x + 3 is (are) **[Raj. NTSE Stage - 2 2017]**

(1) 1 (2) 5 (3) 1, 9 (4) 9

Q.27 If a polynomial $x^4 – 4x^2 + x^3 + 2x + 1$ is divided by x –1, then remainder will be **[Raj. NTSE Stage - 1 2019]**

(1) 0 (2) 1 (3) 9 (4) –1

Q.28 If $x^2 + 4y^2 + 9z^2 – 4xy – 12yz + 6xz = 0$ then **[Raj. NTSE Stage - 1 2019]**

(1) x = 2y – 3z (2) x = y – 3z
(3) 2x = y – 3z (4) x = 3y – 2z

Q.29 If the polynomial $x^4 – 6x^3 + 16x^2 – 25x + 10$ is divided by another polnomial $x^2–2x + k$, the remainder comes out to be x + a, then the value of a is **[NTSE Stage - 2 2018]**

(1) –1 (2) –5 (3) 1 (4) 5

Q.30 If any polynomial f(x) is divided by $x^2 – 9$, then remainder is 3x + 2. If it is divided by (x – 3) the remainder will be : **(Chhattisgarh NTSE Stage-1 2019)**

(1) – 7 (2) 7 (3) 11 (4) – 11

Q.31 If $2019^x + 2019^{-x} = 3$, then the value of $\sqrt{\frac{2019^{6x}-2019^{-6x}}{2019^{x}-2019^{-x}}}$ **(Delhi NTSE Stage-1 2019)**

(1) 3 (2) 6 (3) 9 (4) 12

Q.32 If $x^4 – 83x^2 + 1 = 0$, then a value of $x^3 – x–3$ is: **[Haryana NTSE Stage-1 2020]**

(1) 758 (2) 756 (3) 739 (4) 737

Q.33 When $x^{100} – 2x^{51} +1$ is divided by $x^2 – 1$, the remainder is r(x). The value of r(–2) + r(2) is: **[Haryana NTSE Stage-1 2020]**

(1) 0 (2) 4 (3) 6 (4) 8

Q.34 If polynomials $3x^3 + x^2 – 4x + P$ and $2x^3 + Px^2 + 3x – 3$ are divided by (x – 2) then get the same remainder. What will be the value of P. **(NTSE 2021 –Rajasthan)**

(1) +3 (2) $\frac{1}{3}$ (3) $-\frac{1}{3}$ (4) -3

Q.35 If ax^3+bx+c is divisible by x^2+dx+1, then : **[Haryana NTSE Stage-1 2020-21]**

(1) $a^2 + b^2 = ac$ (2) $a^2 – c^2 = ab$
(3) $a^2 – b^2 = ac$ (4) $a^2 + c^2 = abfg$

Q.36 If $x^2 – 3x + 1 = 0$, then what is the value of $(x^5 + x^{–5})$? **[Haryana NTSE Stage-1 2020-21]**

(1) 119 (2) 122 (3) 123 (4) 125

Q.37 If $\sqrt{x^2+\sqrt[3]{x^2y^2}}+\sqrt{y^2+\sqrt[3]{x^2y^4}}=k$, then which of the following is true ? **[Haryana NTSE Stage-1 2020-21]**

(1) $x^2 + y^2 = k^2$ (2) $x^{3/2} + y^{3/2} = k^{3/2}$
(3) $x^{2/3} + y^{2/3} = k^{2/3}$ (4) $x^{1/3} + y^{1/3} = k^{1/3}$

ANSWER KEY

SUBJECTIVE

Q.1 (i), (ii), (iii), (iv)

Q.2 (i) 3, (ii) – 2, (iii) 0, (iv) 3/4

Q.3 (i) 2, (ii) 3, (iii) 2, (iv) 4

Q.4 (i) Cubic, (ii) Quadratic,
(iii) Quadratic, (iv) Linear, (v) Cubic

Q.5 (i) 9, (ii) $4+\sqrt{11}$, (iii) $4a^4 + 5a^3 - a^2 + 6$

Q.6 (i) 4, (ii) –1/2, (iii) –1, –2,
(iv) 1, 2, 3 (v) 0, (vi) –s/r

Q.7 (i) Yes (ii) Yes, both
(iii) x = 0 is zero, x = 1 is not zero
(vi) $x = -\frac{1}{\sqrt{3}}$ is zero, $x = \frac{2}{\sqrt{3}}$ is not zero,
(v) Yes, both

Q.10 (i) 4, (ii) –5, (iii) –2, (iv) 7

Q.12 –2

Q.13 – 7

Q.15 (i) $(x + 1)^2$
(ii) (x – 6a) (x – 3a)
(iii) (x – z) (2y – 1)
(iv) $(p + 3p)\,(p - 3q)\,(p^2 + 8q^2)$
(v) $(x-\sqrt{2})(7\sqrt{2}x+4)$
(vi) $(x^2 - 2y + 3z)\,(x^2 + 2y - 3z)$
(vii) $(2\sqrt{3}x-5y)(12x^2+10\sqrt{3}xy+25y^2)$
(viii) $a^4x^4\,(a - x)\,(a + x)$
(ix) 15(x – y) (5y– 3z) (3z – 5x)
(x) 3(x – y) (y – z) (z – x)

Q.16 (x – 4)

Q.17 (i) $\sqrt{2}a+\sqrt{3}b$ (ii) $(\sqrt{2}x+1)(x+\sqrt{2})$
(iii) (6x + 5)

Q.18 (i) 10815, (ii) 9702, (iii) 9880,
(iv) 1030301, (v) 63521199

Q.19 (i) $\frac{27}{8}x^3+\frac{27}{4}x^2+\frac{9}{2}x+1$
(ii) $x^3-2x^2y+\frac{4xy^2}{3}-\frac{8}{27}y^3$

Q.20 (i) $81x^2 + 4y^2 + z^2 + 36xy + 4yz + 18zx$,
(ii) $9x^2 + 4y^2 + z^2 - 12xy + 4yz - 6xz$

Q.21 (i) $x^3 + 27$, (ii) $125a^3 - 27b^3$

Q.28 2

Q.30 m = 2, n = – 1

Q.32 324

Q.33 32

Q.34 (i) $8a^3$, (ii) $8a^2 + 2b^2 + 2c^2 + 4bc$

Q.35 (i) 0, (ii) 0, (iii) 0, (iv) –8700

Q.37 0

Q.38 k = – 3, zeros are –3, –6 & 5

Q.40 8cm, 6cm, 10cm

Q.41 (a) $-x^3 + 12x^2 + 47x + 36$,
(b) 1 feet

ANALYTICAL QUESTIONS

BASIC LEVEL

Que.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
Ans.	3	1	1	1	2	2	3	3	3	4	1	4	4	1	4
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	1	2	2	2	3	2	1	1	2	1	3	4	2	1	1
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**
Ans.	2	1	2	1	3	2	3	3	1	4	2	1	3	1	1
Que.	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Ans.	3	4	4	3	3	4	1	1	2	4	3	4	4	1	4
Que.	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
Ans.	1	2	3	2	3	4	3	4	1	3	3	1	3	2	3
Que.	**76**	**77**	**78**	**79**	**80**	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**
Ans.	4	2	2	2	1	1	3	2	1	2	4	3	3	3	3
Que.	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**	**101**	**102**	**103**	**104**	**105**
Ans.	4	1	1	3	4	3	1	3	1	1	1	4	2	2	3

ADVANCE LEVEL

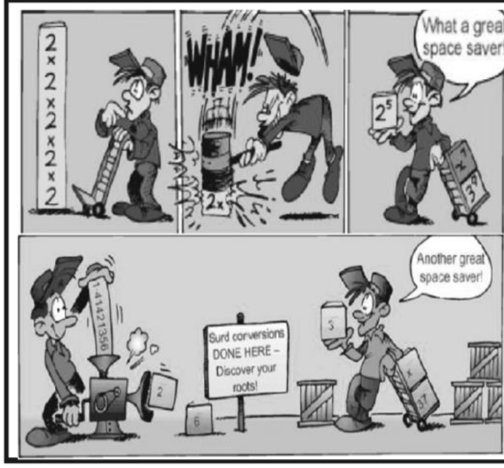
Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	2	1	4	1	1	1	3	4	4	1	2	3	4	3	4
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	3	4	3	1	1	1	3	1	2	4	1	3	3	2	2
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	3	2	4	1	4	2	1	1	3	2	2	2	1	3	1
Que.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	3	3	2	4	2	4	1	3	2	4	4	3	1	3	2
Que.	61	62	63	64											
Ans.	4	2	2	4											

PREVIOUS YEAR QUESTIONS

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	2	4	1	1	3	3	4	4	4	4	1	2	4	1	3
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	3	1	1	1	1	2	3	1	1	3	3	1	2	4	3
Que.	31	32	33	34	35	36	37								
Ans.	4	2	2	2	2	3	3								

Chapter-03

Surds and Indices



The index of a number says how many times to use the number in a multiplication. It is written as a small number to the right and above the base number.

A number that can't be simplified to remove a square root (or cube root etc) is known as surd.
Ex. $\sqrt{2}, \sqrt{3}, \sqrt[3]{11}$

4.1 INTRODUCTION

Let **a** be a rational number and **n** be a positive integer, then irrational number is of the form $\sqrt[n]{a}$ is given a special name **surd**, where 'a' is called **radicand** and it $\sqrt[n]{\ }$ is called the **radical sign** and the index **n** is called **order** of the surd. $\sqrt[n]{a}$ is read as '**n^{th} root of a**' and can also be written as **$a^{1/n}$**.

4.2 LAWS OF SURDS

(i) $(\sqrt[n]{a})^n = (a^{\frac{1}{n}})^n = a^{\frac{1}{n}\times n} = a.$ (ii) $\sqrt[n]{a}.\sqrt[n]{b} = \sqrt[n]{ab}$

(iii) $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$ (iv) $(\sqrt[n]{a})^m = \sqrt[n]{a^m} = a^{m/n}$

(v) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$ (vi) $\sqrt[n]{a} = \sqrt[n\times p]{a^p}$ [Important for changing order of surds] or, $\sqrt[n]{a^m} = \sqrt[n\times p]{a^{m\times p}}$

EX.1 : Simplify : $\sqrt[3]{2}.\sqrt[3]{4}$

Sol. $\sqrt[3]{2}.\sqrt[3]{4} = \sqrt[3]{2\times 4} = \sqrt[3]{2^3} = (2^3)^{1/3} = 2.$

4.2.1 Operation of Surds

(a) Addition and Subtraction of Surds : Addition and subtraction of surds are possible only when order and radicand are same i.e. only for like surds.

EX.2 : Simplify : $5\sqrt[3]{250} + 7\sqrt[3]{16} - 14\sqrt[3]{54}$

Sol. $5\sqrt[3]{250} + 7\sqrt[3]{16} - 14\sqrt[3]{54}$

$= 5\sqrt[3]{125\times 2} + 7\sqrt[3]{8\times 2} - 14\sqrt[3]{27\times 2} = 5\times 5\sqrt[3]{2} + 7\times 2\sqrt[3]{2} - 14\times 3\times\sqrt[3]{2} = (25+14-42)\sqrt[3]{2} = -3\sqrt[3]{2}.$

(b) Multiplication and Division of Surds :

EX.3 : Simplify : $\sqrt{8a^5b}\times\sqrt[3]{4a^2b^2}$.

Sol. $\sqrt{8a^5b}\times\sqrt[3]{4a^2b^2} = \sqrt[8]{8^3a^{15}b^3}\times\sqrt[6]{4^2a^4b^4} = \sqrt[6]{2^{13}a^{19}b^7} = 2^2a^3b\sqrt[6]{2ab} = 4a^3b\sqrt[6]{2ab}$.

EX.4 : Divide : $\sqrt{24}\div\sqrt[3]{200}$.

Sol. $\sqrt{24}\div\sqrt[3]{200} = \dfrac{\sqrt{24}}{\sqrt[3]{200}} = \dfrac{\sqrt[6]{(24)^3}}{\sqrt[6]{(200)^2}} = \sqrt[6]{\dfrac{216}{625}}$.

(c) Comparison of Surds : It is clear that if $x > y > 0$ and $n > 1$ is a positive integer then $\sqrt[n]{x} > \sqrt[n]{y}$.

EX.5 : Arrange $\sqrt{2}, \sqrt[3]{3}$ and $\sqrt[4]{5}$ in ascending order.

Sol. $\sqrt{2}, \sqrt[3]{3}$ and $\sqrt[4]{5}$

L.C.M. of 2, 3, 4 is 12.

$\therefore \sqrt{2} = \sqrt[2\times 6]{2^6} = \sqrt[12]{64}$, $\sqrt[3]{3} = \sqrt[3\times 4]{3^4} = \sqrt[12]{81}$, $\sqrt[4]{5} = \sqrt[4\times 3]{5^3} = \sqrt[12]{125}$

As, 64 < 81 < 125

$\therefore \sqrt[12]{64} < \sqrt[12]{81} < \sqrt[12]{125} \quad \Rightarrow \quad \sqrt{2} < \sqrt[3]{3} < \sqrt[4]{5}$.

4.3 RATIONALIZATION OF SURDS

4.3.1 Rationalizing factor

If the product of two surds is a rational number, then each surd is called a rationalising factor (RF) of the other.

- **Rationalisation of surds :** The process of converting a surd into rational number by multiplying it with a suitable **RF**, is called the rationalisation of the surd.

- **Monomial surds and their RF :** The general form of a monomial surd is $\sqrt[n]{a}$ and its RF is $\mathbf{a^{1-\frac{1}{n}}}$.

EX.6 : Find rationalisation factor of .

Sol. Rationalisation factor of $\sqrt[3]{5}$ is $5^{1-\frac{1}{3}} = 5^{\frac{2}{3}} = \sqrt[3]{5^2} = \sqrt[3]{25}$.

- **Binomial Surds and their RF :** The surds of the types $a+\sqrt{b}$, $a-\sqrt{b}$, $\sqrt{a}+\sqrt{b}$, : and $\sqrt{a}-\sqrt{b}$ are called binomial surds.

- **Conjugate Surds :** The binomial surds which differ only in sign between the terms separating them are known as conjugate surds. In binomial surds, the conjugate surds are RF of each other.

Ex. (i) RF of $\sqrt{a}+\sqrt{b}$ is $\sqrt{a}-\sqrt{b}$. (ii) RF $\sqrt{a}-\sqrt{b}$ of $\sqrt{a}+\sqrt{b}$ is .

EX.7 : Rationalize the denominator $\dfrac{1}{7+5\sqrt{3}}$.

Sol. $\dfrac{1}{7+5\sqrt{3}} = \dfrac{1}{7+5\sqrt{3}}\times\dfrac{7-5\sqrt{3}}{7-5\sqrt{3}} = \dfrac{7-5\sqrt{3}}{49-75} = \dfrac{7-5\sqrt{3}}{-26} = \dfrac{5\sqrt{3}-7}{26}$.

- **Trinomial Surds :** A surd which consists of three terms, atleast two of which are monomial surds, is called a trinomial surd.

Ex. $7+\sqrt{3}+\sqrt{5}$

In order to radionalize = $\dfrac{x}{\sqrt{a}+\sqrt{b}+\sqrt{c}}$

(i) Multiply and divide by $\sqrt{a}+\sqrt{b}-\sqrt{c}$

(ii) Multiply and divide by $(a + b - c) - 2\sqrt{ab}$

EX.8 : Rationalize : $\dfrac{1}{\sqrt{6}+\sqrt{3}+\sqrt{5}}$

Sol. $$\frac{1}{\sqrt{6}+\sqrt{3}+\sqrt{5}}=\frac{1}{(\sqrt{6}+\sqrt{3})+\sqrt{5}}\times\frac{(\sqrt{6}+\sqrt{3})-\sqrt{5}}{(\sqrt{6}+\sqrt{3})-\sqrt{5}}=\frac{\sqrt{6}+\sqrt{3}-\sqrt{5}}{(\sqrt{6}+\sqrt{3})^2-(\sqrt{5})^2}=\frac{\sqrt{6}+\sqrt{3}-\sqrt{5}}{6+3+2\sqrt{18}-5}$$

$$=\frac{\sqrt{6}+\sqrt{3}-\sqrt{5}}{4+6\sqrt{2}}=\frac{\sqrt{6}+\sqrt{3}-\sqrt{5}}{(4+6\sqrt{2})}\times\frac{4-6\sqrt{2}}{4-6\sqrt{2}}=\frac{4\sqrt{6}+4\sqrt{3}-4\sqrt{5}-6\sqrt{12}+6\sqrt{10}-6\sqrt{6}}{-56}$$

$$=\frac{-2\sqrt{6}+4\sqrt{3}-4\sqrt{5}-12\sqrt{3}+6\sqrt{10}}{-56}=\frac{\sqrt{6}+4\sqrt{3}+2\sqrt{5}-3\sqrt{10}}{28}$$

4.4 EXPONENTS (INDEX OR POWER)

The repeated multiplication of the same factor can be written in a more compact form, called exponential form.

4.4.1 Laws of exponents

If 'a' is any non-zero rational number and m, n are whole numbers, then

(i) On the same base in multiplication, powers are added.

$a^m \times a^n = a^{m+n}$; **Ex.** $3^2 \times 3^4 = 3^{2+4} = 3^6$.

(ii) On the same base in division, powers are subtracted.

$\dfrac{a^m}{a^n}=a^{m-n}$ **Ex.** $\dfrac{3^5}{3^2}=3^{5-2}=3^3$.

(iii) $\dfrac{a^m}{a^n}=\dfrac{1}{a^{n-m}}, n>m.$ **Ex.** $\dfrac{2^3}{2^4}=\dfrac{1}{2^{4-3}}=\dfrac{1}{2}$.

(iv) $(a^m)^n = a^{mn}$ **Ex.** $(2^2)^3 = 2^{2\times3} = 2^6$.

(v) $a^n \times a^{-n} = a^0 = 1$

(vi) $a^m \times b^m = (ab)^m$ **Ex.** $2^2 \times 3^2 = (2 \times 3)^2 = 6^2 = 36$.

(vii) $a^{bn} = a^{b+b+b\ \ldots n \text{ times}}$ where a, b are positive real numbers and m, n are rational numbers.

(ix) $(\sqrt[n]{a})^n=a,$ where 'n' is a positive integer and 'a' is a positive rational number.

(x) $\sqrt[n]{a}\sqrt[n]{b}=\sqrt[n]{ab},$ where 'n' is a positive integer and 'a', 'b' are rational numbers.

(xi) $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\dfrac{a}{b}},$ where 'n' is a positive integer and 'a', 'b' are rational numbers.

(xii) $\sqrt[m]{\sqrt[n]{a}}=\sqrt[mn]{a}=\sqrt[n]{\sqrt[m]{a}},$ where 'm', 'n' are positive integer and 'a' is a positive rational number.

(xiii) $\sqrt[n]{\sqrt[m]{(a^k)^m}}=\sqrt[n]{a^k}=\sqrt[mn]{a^{km}}$ where 'm', 'n' are 'k' are positive integers and 'a' is a positive rational number.

(xiv) $\sqrt{a}\times\sqrt{a}=a$

(xv) $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$

(xvi) $(\sqrt{a}+\sqrt{b})^2=a+b+2\sqrt{ab}$

(xvii) $(\sqrt{a}-\sqrt{b})^2=a+b-2\sqrt{ab}$

(xviii) If $a+\sqrt{b}=c+\sqrt{d}\Rightarrow$ a = c and b = d. (equating rational parts and irrational parts)

(xix) $\dfrac{1}{\sqrt{a}-\sqrt{b}}=\dfrac{\sqrt{a}+\sqrt{b}}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})}=\dfrac{\sqrt{a}+\sqrt{b}}{a-b}$

(xx) $\dfrac{1}{\sqrt{a}+\sqrt{b}}=\dfrac{\sqrt{a}-\sqrt{b}}{(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})}=\dfrac{\sqrt{a}-\sqrt{b}}{a-b}$

(xxi) If x = n (n+ 1), then

(a) $\sqrt{x-\sqrt{x-\sqrt{x-.....\infty}}}=n$

(b) $\sqrt{x+\sqrt{x+\sqrt{x+.....\infty}}}=(n+1)$

EX.9 : Simplify : . $\dfrac{(25)^{3/2}\times(243)^{3/5}}{(16)^{5/4}\times(8)^{4/3}}$

Sol. We have, $\dfrac{(25)^{3/2}\times(243)^{3/5}}{(16)^{5/4}\times(8)^{4/3}}=\dfrac{(5^2)^{3/2}\times(3^5)^{3/5}}{(2^4)^{5/4}\times(2^3)^{4/3}}=\dfrac{5^{2\times3/2}\times3^{5\times3/5}}{2^{4\times5/4}\times2^{3\times4/3}}=\dfrac{5^3\times3^3}{2^5\times2^4}=\dfrac{125\times27}{32\times16}=\dfrac{3375}{512}$.

EX.10 : If $25^{x-1}=5^{2x-1}-100$, find the value of x.

Sol. We have, $25^{x-1}=5^{2x-1}-100$

$\Rightarrow (5^2)^{x-1}=5^{2x-1}-100$ $\Rightarrow 5^{2x-2}-5^{2x-1}=-100$ $\Rightarrow 5^{2x-2}-5^{2x-2}.5^1=-100$

$\Rightarrow 5^{2x-2}(1-5)=-100$ $\Rightarrow 5^{2x-2}(-4)=-100$ $\Rightarrow 5^{2x-2}=25$

$\Rightarrow 5^{2x-2}=5^2$ $\Rightarrow 2x-2=2$ $\Rightarrow 2x=4$ $\Rightarrow x=2.$

NEET Sarthi KOTA

ANALYTICAL QUESTIONS

Q.1 The expression $(\sqrt{5}-\sqrt{3})\ (\sqrt{7}-\sqrt{2})$ when simplified becomes a
(1) simple surd (2) mixed surd
(3) compound surd (4) binomial surd

Q.2 The surds $\sqrt{2}, \sqrt[3]{3}$ and $\sqrt[5]{5}$, in their descending order are
(1) $\sqrt[3]{3}, \sqrt[5]{5}, \sqrt{2}$ (2) $\sqrt{2}, \sqrt[3]{3}, \sqrt[5]{5}$
(3) $\sqrt{2}, \sqrt[5]{5}, \sqrt[3]{3}$ (4) $\sqrt[3]{3}, \sqrt{2}, \sqrt[5]{5}$

Q.3 The smallest among the surds $\sqrt{10}-\sqrt{5}, \sqrt{19}-\sqrt{14}, \sqrt{22}-\sqrt{17}$ and $\sqrt{8}-\sqrt{3}$ is
(1) $\sqrt{10}-\sqrt{5}$ (2) $\sqrt{19}-\sqrt{14}$
(3) $\sqrt{22}-\sqrt{17}$ (4) $\sqrt{8}-\sqrt{3}$

Q.4 If the surds $\sqrt[4]{4}, \sqrt[6]{5}, \sqrt[8]{6}$ and $\sqrt[12]{8}$ are arranged in ascending order from left to right, then the third surd from the left is
(1) $\sqrt[12]{8}$ (2) $\sqrt[4]{4}$
(3) $\sqrt[8]{6}$ (4) $\sqrt[6]{5}$

Q.5 $8^{2/3}$ is equal to :
(1) 11/2 (2) 64/3
(3) 4 (4) 7/2

Q.6 $16^{3/4}$ is equal to :
(1) $4\sqrt{2}$ (2) 8 (3) $2\sqrt{2}$ (4) 16

Q.7 $(0.01024)^{1/5}$ is equal to :
(1) 4.0 (2) 0.04
(3) 0.4 (4) 0.00004

Q.8 $(64)^{-2/3} \times (1/4)^{-2}$ is equal to :
(1) 1 (2) 2 (3) 1 / 2 (4) 1 / 16

Q.9 $(36)^{1/6}$ is equal to :
(1) 1 (2) 6 (3) $\sqrt{6}$ (4) $\sqrt[3]{6}$

Q.10 $\left(\dfrac{8}{125}\right)^{-(4/3)}$ is simplifies to :
(1) 625 / 16 (2) 625 / 8
(3) 625 / 32 (4) 16 / 625

Q.11 The value of $\sqrt{2^4} + \sqrt[3]{64} + \sqrt[2]{2^4}$ is :
(1) 12 (2) 16
(3) 18 (4) 24

Q.12 Which one of the following is the least $\sqrt{3}, \sqrt[3]{2}, \sqrt{2}$ and $\sqrt[3]{4}$?
(1) $\sqrt{2}$ (2) $\sqrt[3]{2}$
(3) $\sqrt{3}$ (4) $\sqrt[3]{3}$

Q.13 Which one of the following is the biggest $\sqrt[3]{4}, \sqrt[4]{6},$ and $\sqrt[12]{245}$?
(1) $\sqrt[3]{4}$ (2) $\sqrt[4]{6}$
(3) $\sqrt[6]{15}$ (4) $\sqrt[12]{245}$

Q.14 Which of the following number is the least $(0.5)^2, \sqrt{0.49}, \sqrt[3]{0.008}, 0.23$?
(1) $(0.5)^2$ (2) $\sqrt{0.49}$
(3) $\sqrt[3]{0.008}$ (4) 0.23

Q.15 Arrange of the following in descending order : $\sqrt[3]{4}, \sqrt{2}, \sqrt[6]{3}, \sqrt[4]{5}$
(1) $\sqrt[3]{4} > \sqrt[4]{5} > \sqrt{2} > \sqrt[6]{3}$
(2) $\sqrt[3]{5} > \sqrt[3]{4} > \sqrt[6]{3} > \sqrt{2}$
(3) $\sqrt{2} > \sqrt[4]{5} > \sqrt[3]{4} > \sqrt{2}$
(4) $\sqrt{2} > \sqrt[4]{5} > \sqrt[3]{4} > \sqrt{2}$

Q.16 The greatest of the numbers $(2.89)^{0.5}$, $2-(0.5)^2$, $1 + \dfrac{0.5}{1-\frac{1}{2}}, \sqrt{3}$
(1) $(2.89)^{0.5}$ (2) $2-(0.5)^2$
(3) 1+0.5 / 1–(1/2) (4) $\sqrt{3}$

Q.17 The approximate value of $\dfrac{3\sqrt{12}}{2\sqrt{28}} \div \dfrac{2\sqrt{21}}{\sqrt{98}}$ is
(1) 1.0727 (2) 1.0606
(3) 1.6026 (4) 1.6007

Q.18 $[\sqrt[3]{2} \times \sqrt{2} \times \sqrt[3]{3} \times \sqrt{3}]$ is equal to
(1) 6^5 (2) $6^{5/6}$
(3) 6 (4) None of these

Q.19 $(6.5 \times 6.5 - 45.5 + 3.5 \times 3.5)$ is equal to :
(1) 10 (2) 9 (3) 7 (4) 6

Q.20 The value of $(243)^{0.16} \times (243)^{0.04}$ is equal to :
(1) 0.16 (2) 3
(3) 1/3 (4) 0.04

Q.21 $2\sqrt[3]{32} - 3\sqrt[3]{4} + \sqrt[3]{500}$ is equal to :
(1) $4\sqrt[3]{6}$ (2) $3\sqrt{24}$
(3) $6\sqrt[3]{4}$ (4) 916

Q.22 $(16^{0.16} \times 2^{0.36})$ is equal to :
(1) 2 (2) 16 (3) 32 (4) 64

Q.23 $\dfrac{3^0 + 3^{-1}}{3^0 - 3^{-1}}$ is simplified to
(1) –2 (2) – 1
(3) 1 (4) 2

Q.24 $\sqrt{\sqrt[3]{0.004096}}$ is equal to
(1) 4 (2) 0.4
(3) 0.04 (4) 0.004

Q.25 $\{(-2)^{-2}\}^{-2}$ is equal to :
(1) 16 (2) 8
(3) – 8 (4) –1

Q.26 $[3 - 4(3 - 4) - 1]^{-1}$ is equal to :
(1) 7 (2) – 7
(3) 1/6 (4) –(1/7)

Q.27 The smallest of $\sqrt{8}+\sqrt{5}, \sqrt{7}+\sqrt{6}, \sqrt{10}+\sqrt{3}$ and $\sqrt{11}+\sqrt{2}$ is :
(1) $\sqrt{8}+\sqrt{5}$ (2) $\sqrt{7}+\sqrt{6}$
(3) $\sqrt{10}+\sqrt{3}$ (4) $\sqrt{11}+\sqrt{2}$

Q.28 $\sqrt{8-2\sqrt{15}}$ is equal to :
(1) $\sqrt{5}+\sqrt{3}$ (2) $5-\sqrt{3}$
(3) $\sqrt{5}-\sqrt{3}$ (4) $3-\sqrt{5}$

Q.29 $(0.04)^{-(1.5)}$ is equal to
(1) 25 (2) 125 (3) 60 (4) 5

Q.30 $(\sqrt{2}+\sqrt{7-2\sqrt{10}})$ is equal to
(1) $\sqrt{2}$ (2) $\sqrt{7}$ (3) $\sqrt{5}$ (4) $2\sqrt{5}$

Q.31 By how much does $(\sqrt{12}+\sqrt{18})$ exceed $(2\sqrt{3}+2\sqrt{2})$?
(1) 2 (2) $\sqrt{3}$ (3) $\sqrt{2}$ (4) 3

Q.32 The value of $\sqrt[3]{1372} \times \sqrt[3]{1458} \div \sqrt[3]{343}$ is
(1) 18 (2) 15
(3) 13 (4) 12

Q.33 Evaluate $16\sqrt{\dfrac{3}{4} - 9\sqrt{\dfrac{4}{3}}}$ if $\sqrt{12} = 3.46$
(1) 3.46 (2) 10.38
(3) 13.84 (4) 24.22

Q.34 $\left[\left\{\left(-\dfrac{1}{2}\right)^2\right\}^{-2}\right]^{-1}$ is equal to
(1) 1/16 (2) 16
(3) – (1/16) (4) – 16

Q.35 Simplified form of $\left[(\sqrt[5]{x^{-3/5}})^{-5/3}\right]^{-5}$ is
(1) x^5 (2) x^{-5}
(3) x (4) 1/x

Q.36 The number which when multiplied with $(\sqrt{3}+\sqrt{2})$ gives $(\sqrt{12}+\sqrt{18})$ is
(1) $3\sqrt{2}-2\sqrt{3}$ (2) $3\sqrt{2}+2\sqrt{3}$
(3) $\sqrt{6}$ (4) $2\sqrt{3}-3\sqrt{2}$

Q.37 The square root of $14+6\sqrt{5}$
(1) $2+\sqrt{5}$ (2) $3+\sqrt{5}$
(3) $5+\sqrt{3}$ (4) $3+2\sqrt{5}$

Q.38 Among the numbers $\sqrt{2}, \sqrt[3]{9}, \sqrt[4]{16}, \sqrt[5]{32}$ the greatest one is
(1) $\sqrt{2}$ (2) $\sqrt[3]{9}$
(3) $\sqrt[4]{16}$ (4) $\sqrt[5]{32}$

Q.39 The square root of $\left(\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)$ is
(1) $\sqrt{3}+\sqrt{2}$ (2) $\sqrt{3}-\sqrt{2}$
(3) $\sqrt{2}\pm\sqrt{3}$ (4) $\sqrt{2}-\sqrt{3}$

Q.40 The value of $\sqrt{40+\sqrt{9\sqrt{81}}}$ is

(1) $\sqrt{111}$ (2) 9 (3) 7 (4) 11

Q.41 If $\frac{(x-\sqrt{24})(\sqrt{75}+\sqrt{50})}{\sqrt{75}-\sqrt{50}}$ = 1, then the value of x is

(1) $\sqrt{5}$ (2) 5 (3) $2\sqrt{5}$ (4) v

Q.42 Find the simplest value of $2\sqrt{50}+\sqrt{18}-\sqrt{72}$ (given $\sqrt{2}=1.414$).

(1) 4.242 (2) 9.898
(3) 10.6312 (4) 8.484

Q.43 $2\sqrt[3]{40}-4\sqrt[3]{320}+3\sqrt[3]{625}-3\sqrt[3]{5}$ is equal to

(1) $-2\sqrt[3]{340}$ (2) 0
(3) $\sqrt[3]{340}$ (4) $\sqrt[3]{660}$

Q.44 Let $\sqrt[3]{a}=\sqrt[3]{26}+\sqrt[3]{7}+\sqrt[3]{63}$ then

(1) a < 729 but a > 216
(2) a < 216
(3) a > 729
(4) a = 729

Q.45 The value of $\sqrt{2^3\sqrt{4\sqrt{2\sqrt[3]{4}}}}$ is

(1) 2 (2) 2^2 (3) 2^3 (4) 2^5

Q.46 The simplified value of $(\sqrt{3}+1)(10+\sqrt{12})(\sqrt{12}-2)(5-\sqrt{3})$ is

(1) 16 (2) 88 (3) 176 (4) 132

Q.47 $\frac{\sqrt{10+\sqrt{25+\sqrt{108+\sqrt{154+\sqrt{225}}}}}}{\sqrt[3]{8}}=?$

(1) 8 (2) 4 (3) 1/2 (4) 2

Q.48 By how much $\sqrt{12}+\sqrt{18}$ does exceed $\sqrt{3}+\sqrt{2}$?

(1) $\left(\sqrt{3}+2\sqrt{2}\right)$ (2) $2(\sqrt{3}+\sqrt{2})$
(3) $(\sqrt{3}+2\sqrt{2})$ (4) $2(\sqrt{3}-2\sqrt{2})$

Q.49 The value of $\sqrt{5+2\sqrt{6}}-\frac{1}{\sqrt{5+2\sqrt{6}}}$ is :

(1) $2\sqrt{2}$ (2) $2\sqrt{3}$
(3) $1+\sqrt{5}$ (4) $\sqrt{5}-1$

Q.50 Simplify : $\left(\frac{\frac{3}{2+\sqrt{3}}-\frac{2}{2-\sqrt{3}}}{2-5\sqrt{3}}\right)$

(1) $1/2-5\sqrt{3}$ (2) $2-5\sqrt{3}$
(3) 1 (4) 0

Q.51 equals : $(\sqrt{8}-\sqrt{4}-\sqrt{2})$

(1) $2-\sqrt{2}$ (2) $\sqrt{2}-2$
(3) 2 (4) – 2

Q.52 Simplify : $\left[\sqrt[3]{\sqrt[6]{5^9}}\right]^4\left[\sqrt[3]{\sqrt[6]{5^9}}\right]^4$

(1) 5^2 (2) 5^4 (3) 5^8 (4) 5^{12}

Q.53 The simplified form of $(16^{3/2} + 16^{-3/2})$ is :

(1) 0 (2) 4097 / 64
(3) 1 (4) 16 / 4097

Q.54 $\left(\frac{2+\sqrt{3}}{2-\sqrt{3}}+\frac{2-\sqrt{3}}{2+\sqrt{3}}+\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$ simplifies to :

(1) $2-\sqrt{3}$ (2) $2+\sqrt{3}$
(3) $16-\sqrt{3}$ (4) $40-\sqrt{3}$

Q.55 $\left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)^2+\left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)^2$ is equal to :

(1) 64 (2) 62 (3) 66 (4) 68

Q.56 The value of $\sqrt{\frac{(\sqrt{12}-\sqrt{8})(\sqrt{3}+\sqrt{2})}{5+\sqrt{24}}}$ is :

(1) $\sqrt{6}-\sqrt{2}$ (2) $\sqrt{6}+\sqrt{2}$
(3) $\sqrt{6}-2$ (4) $2-\sqrt{6}$

Q.57 The value of $\sqrt{11+2\sqrt{30}}-\frac{1}{\sqrt{11+2\sqrt{30}}}$ is :

(1) $2\sqrt{5}$ (2) $2\sqrt{6}$ (3) $1+\sqrt{6}$ (4) $1+\sqrt{5}$

Q.58 Given $\sqrt{2}=1.414$. The value of $\sqrt{8}+2\sqrt{32}-3\sqrt{128}+4\sqrt{50}$ is

(1) 8.484 (2) 8.526
(3) 8.426 (4) 8.876

Q.59 If $\sqrt{15}=3.88$, then what is the value of $\sqrt{\frac{5}{3}}$?

(1) 1.293 (2) 1.2934
(3) 1.29 (4) 1.295

Q.60 The value of 2 + $\sqrt{0.09}-\sqrt[3]{0.008}-75\%$ of 2.80 is :

(1) 0 (2) 0.01
(3) – 1 (4) 0.001

Q.61 The value of $(3+2\sqrt{2})^{-3}+(3-2\sqrt{2})^{-3}$ is

(1) 189 (2) 180 (3) 108 (4) 198

Q.62 $\frac{\sqrt{5}}{\sqrt{3}+\sqrt{2}}-\frac{3\sqrt{3}}{\sqrt{5}+\sqrt{2}}+\frac{2\sqrt{2}}{\sqrt{5}+\sqrt{3}}$ is equal to :

(1) 0 (2) $2\sqrt{15}$
(3) $2\sqrt{10}$ (4) $2\sqrt{6}$

Q.63 The value of

$(\sqrt[3]{3.5}+\sqrt[3]{2.5})\{(\sqrt[3]{3.5})^2-\sqrt[3]{8.75}+(\sqrt[3]{2.5})^2\}$

is :

(1) 5.375 (2) 1
(3) 6 (4) 5

Q.64 The value of $\frac{1}{\sqrt{3.25}+\sqrt{2.25}}+\frac{1}{\sqrt{4.25}+\sqrt{3.25}}+\frac{1}{\sqrt{5.25}+\sqrt{4.25}}+\frac{1}{\sqrt{6.25}+\sqrt{5.25}}$ is :

(1) 1.00 (2) 1.25
(3) 1.50 (4) 2.25

Q.65 $\frac{10.3\times10.3\times10.3+1}{10.3\times10.3-10.3+1}$ is equal to

(1) 9.3 (2) 10.3 (3) 11.3 (4) 12.3

Q.66 When $(4+\sqrt{7})$ is presented in the form of perfect square it will be equal to :

(1) $(2+\sqrt{7})^2$ (2) $\left(\frac{\sqrt{7}}{2}+\frac{1}{2}\right)^2$

(3) $\left\{\frac{1}{\sqrt{2}}(\sqrt{7}+1)\right\}^2$ (4) $(\sqrt{3}+4)^2$

Q.67 The simplified form of

$\frac{2}{\sqrt{7}+\sqrt{5}}+\frac{7}{\sqrt{12}-\sqrt{5}}-\frac{5}{\sqrt{12}-\sqrt{7}}$

is :

(1) 5 (2) 2 (3) 1 (4) 0

Q.68 $\frac{1}{\sqrt{3}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{9}}$ is

(1) $\sqrt{3}$ (2) $3\sqrt{3}$
(3) $3-\sqrt{3}$ (4) $5-\sqrt{3}$

Q.69 Simplify:

$\frac{1}{\sqrt{100}-\sqrt{99}}-\frac{1}{\sqrt{99}-\sqrt{98}}+\frac{1}{\sqrt{98}-\sqrt{97}}-\frac{1}{\sqrt{97}-\sqrt{96}}+....+\frac{1}{\sqrt{2}-\sqrt{1}}$

(1) 10 (2) 9 (3) 13 (4) 11

Q.70 $\left[\frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}}+\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}}\right]$ in simplified form equals to

(1) 1 (2) $\sqrt{2}$
(3) $\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}$ (4) 0

Q.71 Which is the greatest among $(\sqrt{19}-\sqrt{17}),(\sqrt{13}-\sqrt{11})$ $(\sqrt{7}-\sqrt{5})$, and $(\sqrt{5}-\sqrt{3})$?

(1) $\sqrt{19}-\sqrt{17}$
(2) $\sqrt{13}-\sqrt{11}$
(3) $\sqrt{7}-\sqrt{5}$
(4) $\sqrt{5}-\sqrt{3}$

Q.72 If x = $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and y = $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ then (x + y)=?

(1) 8 (2) 16
(3) $2\sqrt{15}$ (4) $2(\sqrt{5}+\sqrt{3})$

Q.73 $\frac{12}{3+\sqrt{5}+2\sqrt{2}}$ is equal to

(1) $1-\sqrt{5}+\sqrt{2}+\sqrt{16}$

(2) $1+\sqrt{5}+\sqrt{2}-\sqrt{10}$

(3) $1+\sqrt{5}+\sqrt{2}+\sqrt{10}$

(4) $1-\sqrt{5}-\sqrt{2}+\sqrt{10}$

Q.74 $3+\frac{1}{\sqrt{3}}+\frac{1}{3+\sqrt{3}}+\frac{1}{\sqrt{3}-3}$ is equal to

(1) 1 (2) 3

(3) $3+\sqrt{3}$ (4) $3-\sqrt{3}$

Q.75 $\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}}-\frac{2\sqrt{6}}{\sqrt{3}+1}+\frac{2\sqrt{3}}{\sqrt{6}+2}$ is equal to

(1) 3 (2) 2 (3) 0 (4) $\sqrt{3}$

Q.76 Given that $\sqrt{3}=1.732$, the value of $\frac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}}$

(1) 4.899 (2) 2.551

(3) 1.414 (4) 1.732

Q.77 Given that $\sqrt{5}=2.236$ and $\sqrt{3}=1.732$; the value of $\frac{1}{\sqrt{5}+\sqrt{3}}$ is

(1) 0.564 (2) 0.504

(3) 0.253 (4) 0.202

Q.78 If a = $\frac{\sqrt{3}}{2}$, then the value of $\sqrt{1+a}+\sqrt{1-a}$ is :

(1) $\sqrt{3}$ (2) $\sqrt{3}/2$

(3) $2+\sqrt{3}$ (4) $2-\sqrt{3}$

Q.79 If a = $\frac{\sqrt{5}+1}{\sqrt{5}-1}, b=\frac{\sqrt{5}-1}{\sqrt{5}+1}$, the value of $\frac{a^2+ab+b^2}{a^2-ab+b^2}$ is

(1) 3/4 (2) 4/3 (3) 3/5 (4) 5/3

Q.80 $\frac{2}{\sqrt{5}+\sqrt{3}}-\frac{3}{\sqrt{6}-\sqrt{3}}+\frac{1}{\sqrt{6}+\sqrt{5}}$ is equal to

(1) $-2\sqrt{6}$ (2) $-2\sqrt{5}$

(3) $-2\sqrt{3}$ (4) 0

Q.81 $\left[\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}-\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right]$ simplifies to

(1) $2\sqrt{6}$ (2) $4\sqrt{6}$

(3) $2\sqrt{3}$ (4) $3\sqrt{2}$

Q.82 $\frac{1}{\sqrt{9}-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-\sqrt{4}}$ is equal to :

(1) 5 (2) 1 (3) 3 (4) 0

Q.83 Given that $\sqrt{5}=2.24$, then the value of $\frac{3\sqrt{5}}{2\sqrt{5}-0.48}$ is

(1) 0.168 (2) 1.68

(3) 16.8 (4) 168

Q.84 $\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}=?$

(1) 5 (2) 4 (3) 3 (4) 2

Q.85 $\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}$ is equal to

(1) $5+2\sqrt{6}$ (2) $\frac{3+2\sqrt{6}}{2}$

(3) $5-2\sqrt{3}$ (4) $5+2\sqrt{3}$

Q.86 $\frac{\sqrt{3}+1}{\sqrt{3}-1}+\frac{\sqrt{2}+1}{\sqrt{2}-1}+\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{2}-1}{\sqrt{2}+1}$ is simplified to

(1) 10 (2) 12 (3) 14 (4) 18

Q.87 Find the value of x in the expression $\sqrt[4]{3x+1}=2$

(1) 3 (2) 6 (3) 4 (4) 5

Q.88 $\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}+\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}$ is equal to :

(1) 12 (2) $6\sqrt{35}$ (3) 6 (4) $2\sqrt{35}$

Q.89 $\left(\frac{2}{\sqrt{6}+2}+\frac{1}{\sqrt{7}+\sqrt{6}}+\frac{1}{\sqrt{8}-\sqrt{7}}+2-2\sqrt{2}\right)$ is equal to

(1) 0 (2) $2\sqrt{2}$

(3) $\sqrt{2}$ (4) $2\sqrt{7}$

Q.90 The value of $\frac{2+\sqrt{3}}{2-\sqrt{3}}+\frac{2-\sqrt{3}}{2+\sqrt{3}}+\frac{\sqrt{3}+1}{\sqrt{3}-1}$ is

(1) $16+\sqrt{3}$ (2) $4+\sqrt{3}$

(3) $2-\sqrt{3}$ (4) $2\sqrt{3}$

Q.91 The value of $\frac{1}{\sqrt{2}+1}+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+....$ $+\frac{1}{\sqrt{100}+\sqrt{99}}$ is

(1) 1 (2) 9

(3) $\sqrt{199}$ (4) $\sqrt{99}-1$

Q.92 If $\sqrt{2}=1.4142$, find the value of

$2\sqrt{2}+\sqrt{2}+\frac{1}{2+\sqrt{2}}-\frac{1}{2-\sqrt{2}}$

(1) 1.4144 (2) 2.8284 (3) 28.284 (4) 2.4142

Q.93 The greatest of the following numbers 0.16, $\sqrt{0.16}$, $(0.16)^2$, 0.04 is

(1) 0.16 (2) $\sqrt{0.16}$

(3) 0.04 (4) $(0.16)^2$

Q.94 Evaluate $\sqrt{20}+\sqrt{12}+\sqrt[3]{729}-\frac{4}{\sqrt{5}-\sqrt{3}}-\sqrt{81}$ c

(1) $\sqrt{2}$ (2) $\sqrt{3}$ (3) 0 (4) $2\sqrt{2}$

Q.95 Let a = $\frac{1}{2-\sqrt{3}}+\frac{1}{3-\sqrt{8}}+\frac{1}{4-\sqrt{15}}$ then we have

(1) a < 18 but a ≠ 9

(2) a > 18

(3) a = 18

(4) a = 9

Q.96 equals to $2+\frac{6}{\sqrt{3}}+\frac{1}{2+\sqrt{3}}+\frac{1}{\sqrt{3}-2}$

(1) $+(2\sqrt{3})$ (2) $-(2+\sqrt{3})$

(3) 1 (4) 2

Q.97 If a, b are rationals and $a\sqrt{2}+b\sqrt{3}=\sqrt{98}+\sqrt{108}-\sqrt{48}-\sqrt{72}$ then the value of a, b are respectively.

(1) 1, 2 (2) 1, 3 (3) 2, 1 (4) 2, 3

Q.98 If $\frac{4+3\sqrt{3}}{7+4\sqrt{3}}=A+B\sqrt{3}$ then $B-A$ is

(1) – 13 (2) $2\sqrt{13}$

(3) 13 (4) $3\sqrt{3}-\sqrt{7}$

Q.99 Find the value of $\sqrt{30+\sqrt{30+\sqrt{30+....}}}$

(1) 5 (2) $3\sqrt{10}$

(3) 6 (4) 7

Q.100 The value of $\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$ $-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{3-\sqrt{8}}$ is

(1) 0 (2) 1 (3) 5 (4) 7

Q.101 The simplified value of

$\frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}}+\frac{1}{\sqrt{2}-\sqrt{3}-\sqrt{5}}$

(1) 0 (2) 1

(3) $\sqrt{2}$ (4) $1/\sqrt{2}$

Q.102 The simplified value of

$\frac{\sqrt{6}+2}{\sqrt{2}+\sqrt{2+\sqrt{3}}}-\frac{\sqrt{6}-2}{\sqrt{2}-\sqrt{2-\sqrt{3}}}-\frac{2\sqrt{2}}{2+\sqrt{2}}$

(1) $2\sqrt{6}$ (2) 2 (3) $\sqrt{3}$ (4) 0

Q.103 $\frac{6^2+7^2+8^2+9^2+10^2}{\sqrt{7+4\sqrt{3}}-\sqrt{4+2\sqrt{3}}}$ is equal to

(1) 330 (2) 355

(3) 305 (4) 366

Q.104 The value of $\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}$ $+\frac{1}{\sqrt{5}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{9}}$ is

(1) 2 (2) 4 (3) 0 (4) 1

Q.105 The value of $\sqrt{72+\sqrt{72+\sqrt{72+......}}}$ is

(1) 9 (2) 18 (3) 8 (4) 12

Q.106 If $\sqrt{33}=5.745$, then the value of the following is approximately $\sqrt{33}/11$:

(1) 0.5223 (2) 6.32
(3) 2.035 (4) 1

Q.107 The value of $\frac{1}{1+\sqrt{2}+\sqrt{3}}+\frac{1}{1-\sqrt{2}+\sqrt{3}}$ is:

(1) $\sqrt{2}$ (2) $\sqrt{3}$
(3) 1 (4) $4(\sqrt{3}+\sqrt{2})$

Q.108 The value of $\frac{3\sqrt{7}}{\sqrt{5}+\sqrt{2}}-\frac{5\sqrt{5}}{\sqrt{2}+\sqrt{7}}+\frac{2\sqrt{2}}{\sqrt{7}+\sqrt{5}}$ is:

(1) 1 (2) 0 (3) $2\sqrt{3}$ (4) $\sqrt{7}$

Q.109 If $11\sqrt{n}=\sqrt{112}+\sqrt{343},$ then the value of n is :

(1) 3 (2) 11 (3) 13 (4) 7

Q.110 The exponential form of $\sqrt{\sqrt{2}\times\sqrt{3}}$ is

(1) $6^{-1/2}$ (2) $6^{1/2}$ (3) $6^{1/4}$ (4)6

Q.111 (3x – 2y) : (2x + 3y) = 5 : 6, then one of the value of $\left(\frac{\sqrt[3]{x}+\sqrt[3]{y}}{\sqrt[3]{x}-\sqrt[3]{y}}\right)^2$ is

(1) 1 / 25 (2) 5
(3) 1 / 5 (4) 25

Q.112 A tap is dripping at a constant rate into a container. The level (L cm) of the water in the container is given by the equation $L = 2 - 2^t$, where t is time taken in hours. Then the level of water in the container at the start is

(1) 0 cm (2) 1 cm (3) 2 cm (4) 4 cm

Q.113 If $5\sqrt{5}\times5^3\div5^{3/2}=5^{a/2}$, then the value of a is

(1) 4 (2) 5 (3) 6 (4) 8

Q.114 $4^{61} + 4^{62} + 4^{63} + 4^{64}$ is divisible by

(1) 17 (2) 3 (3) 11 (4) 13

Q.115 $2^{n-1} + 2^{n+1} = 320$, then the value of n is

(1) 6 (2) 8 (3) 5 (4) 7

Q.116 $55^3 + 17^3 - 72^3 + 201960$ is equal to

(1) –1 (2) 0 (3) 1 (4) 17

Q.117 The value of $\frac{(243)^{n/5}\times3^{2n+1}}{9^n\times3^{n-1}}$ is

(1) 3 (2) 9 (3) 6 (4) 12

Q.118 $\frac{0.355\times0.5555\times2.025}{0.225\times1.775\times0.2222}$ is equal to

(1) 5.4 (2) 4.58
(3) 4.5 (4) 5.45

Q.119 If $2^x = 3^y = 6^{-z}$ then $\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ is

(1) 0 (2) 1
(3) 3/2 (4) –(1/2)

Q.120 $\left(\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}\right)$ is simplifies to

(1) $\sqrt{5}+\sqrt{6}$ (2) $2\sqrt{5}+\sqrt{6}$
(3) $\sqrt{5}-\sqrt{6}$ (4) $2\sqrt{5}-3\sqrt{6}$

Q.121 If $0.42 \times 100^k = 42$, then the value of k is

(1) 4 (2) 2 (3) 1 (4) 3

Q.122 $\frac{(0.05)^2+(0.41)^2+(0.073)^2}{(0.005)^2+(0.041)^2+(0.0073)^2}=?$

(1) 10 (2) 100
(3) 1000 (4) None of these

Q.123 $\sqrt{8\sqrt{8\sqrt{8}}}$ is equal to

(1) $8^{\frac{24}{8}}$ (2) $8^{\frac{512}{511}}$
(3) $8^{\frac{511}{512}}$ (4) $8^{\frac{512}{3}}$

Q.124 The greatest number among 2^{60}, 3^{48}, 4^{36} and 5^{24} is

(1) 2^{60} (2) 3^{48}
(3) 4^{36} (4) 5^{24}

Q.125 $\frac{(3.06)^3-(1.98)^3}{(3.06)^2+3.06\times1.98+(1.98)^2}$ is equal to :

(1) 1.08 (2) 5.04
(3) 2.16 (4) 1.92

Q.126 If $3^{x+y} = 81$ and $81^{x-y} = 3$, then the value of x is

(1) 42 (2) 15 / 8
(3) 17 / 8 (4) 39

Q.127 The value of $\frac{(0.337+0.126)^2-(0.337-0.126)^2}{0.337\times0.126}$ is

(1) 0 (2) 1 (3) 4 (4) 2

Q.128 If m and n(n > 1) are whole numbers such that $m^n = 121$, then value of $(m-1)^{n+1}$ is?

(1) 1 (2) 10 (3) 121 (4) 1000

Q.129 If $x + \frac{1}{x} = -2$ then the value of $x^{2n+1} + \frac{1}{x^{2n+1}}$ where n is a positive integer is

(1) 0 (2) 2 (3) – 2 (4) – 5

Q.130 $\frac{(998)^2-(997)^2-45}{(98)^2-(97)^2}$ = ?

(1) 1995 (2) 195 (3) 95 (4) 10

Q.131 $\frac{(5.624)^3-(4.376)^3}{5.624\times5.624-(5.624\times4.376)+4.376\times4.376}$ is equal to

(1) 10 (2) 1.248 (3) 20.44 (4) 1

Q.132 $\sqrt{56-\sqrt{56-\sqrt{56-........}}}$ is equal to

(1) 8 (2) 7 (3) 6 (4) 4

Q.133 $2\sqrt[3]{32}-3\sqrt[3]{4}\div\sqrt[3]{500}$ = ?

(1) $4\sqrt[3]{6}$ (2) $3\sqrt{24}$ (3) $6\sqrt[3]{4}$ (4) 916

Q.134 $\left(\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\frac{1}{10.13}+\frac{1}{13.16}\right)$ is equal to

(1) 1 / 3 (2) 5 / 16
(3) 3 / 8 (4) 41 / 7280

Q.135 $\left[8-\left(\frac{4^{9/2}\sqrt{2.2^2}}{2\sqrt{2^{-2}}}\right)^{1/2}\right]$ is equal to

(1) 32 (2) 8 (3) 1 (4) 0

Q.136 What will be the number of two digits made from the units and tens digits of the expression $2^{12n}-6^{4n}$ where n is a positive integer?

(1) 10 (2) 100 (3) 30 (4) 02

Q.137 $\frac{5.71\times5.71\times5.71-2.79\times2.79\times2.79}{5.71\times5.71+5.71\times2.79+2.79\times2.79}$ in simplified form is :

(1) 8.5 (2) 8.6 (3) 2.82 (4) 2.92

Q.138 $\frac{(2.3)^3+0.027}{(2.3)^2-0.69+0.09}$ is equal to :

(1) 2.60 (2) 2.00 (3) 2.33 (4) 2.80

Q.139 $(16)^{0.16}\times(16)^{0.04}\times(2)^{0.2}$ is equal to :

(1) 1 (2) 2 (3) 4 (4) 16

Q.140 $\frac{(0.96)^3-(0.1)^3}{(0.96)^2+0.096+(0.1)^2}$ is simplified to :

(1) 1.06 (2) 0.95
(3) 0.86 (4) 0.97

Q.141 $(0.04)^{-1.5}$ on simplification gives :

(1) 25 (2) 125 (3) 250 (4) 625

Q.142 The value of $\frac{(243)^{0.13}\times(243)^{0.07}}{(7)^{0.25}\times(49)^{0.075}\times(343)^{0.2}}$ is

(1) 3 / 7 (2) 7 / 3 (3) 10 / 7 (4) 16 / 7

Q.143 If $(125)^{2/3}\times(625)^{-1/4}=(5)^x$, then the value of x is :

(1) 3 (2) 2 (3) 0 (4) 1

Q.144 Simplify: $\frac{0.41\times0.41\times0.41+0.69\times0.69\times0.69}{0.41\times0.41-0.41\times0.69+0.69+0.69}$

(1) 0.28 (2) 1.41 (3) 1.1 (4) 2.8

Q.145 Simplify : $\frac{(625)^{1/2}(0.0144)^{1/2}+1}{(0.027)^{1/3}\times(81)^{1/4}}$

(1) 0.14 (2) 1.4 (3) 1 (4) 4.44

Q.146 Simplify:

$$\frac{(1.5)^3+(4.7)^3+(3.8)^3-3\times1.5\times4.7\times3.8}{(1.5)^2+(4.7)^2+(3.8)^2-1.5\times4.7-4.7\times3.8-3.8\times1.5}$$

(1) 0 (2) 1 (3) 10 (4) 30

Q.147 If $3^{x+8}=27^{2x+1}$ the value of x is :

(1) 7 (2) 3 (3) – 2 (4) 1

Q.148 If $27^{n+1}=(243)^3$ then the value of n

(1) 5 (2) 6 (3) 7 (4) 9

PREVIOUS YEAR QUESTIONS

Q.1 If $a^x = b$, $b^y = c$ and $c^z = a$, then value of xyz is

(Rajasthan NTSE Stage-1 2007)

(1) 1 (2) 0 (3) –1 (4) a + b + c.

Q.2 If $a^x = b$, $b^y = c$ and $c^z = a$, then the value of $x^2y^2z^2$ is ....

[Madhya Pradesh NTSE Stage-1 2013]

(1) $a^2b^2c^2$ (2) 1 (3) 4 (4) $\frac{1}{a^2b^2c^2}$

Q.3 Of the following four numbers the largest is :

(Harayana NTSE Stage-1 2014)

(1) 3^{210} (2) 7^{140} (3) $(17)^{105}$ (4) $(31)^{84}$

Q.4 Among the numbers 2^{250}, 3^{200}, 4^{150} and 5^{100}, the greatest is **(West Bengal NTSE Stage-1 2016)**

(1) 2^{250} (2) 3^{200} (3) 4^{150} (4) 5^{100}

ANSWER KEY

ANALYTICAL QUESTIONS

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	3	4	3	4	3	2	3	1	4	1	1	2	1	3	1
Que.	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Ans.	3	2	2	2	2	3	1	1	2	1	3	4	3	2	3
Que.	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**
Ans.	3	1	1	1	4	3	2	2	1	3	2	2	2	1	1
Que.	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
Ans.	3	4	3	1	3	2	2	2	3	2	3	1	1	1	1
Que.	**61**	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
Ans.	4	1	3	1	3	3	4	3	4	3	4	1	2	2	3
Que.	**76**	**77**	**78**	**79**	**80**	**81**	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**
Ans.	4	3	1	2	3	2	1	2	1	1	1	4	1	4	1
Que.	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**	**101**	**102**	**103**	**104**	**105**
Ans.	2	2	2	3	1	4	1	3	3	3	4	4	1	1	1
Que.	**106**	**107**	**108**	**109**	**110**	**111**	**112**	**113**	**114**	**115**	**116**	**117**	**118**	**119**	**120**
Ans.	1	3	2	4	3	4	2	1	1	4	2	2	3	1	3
Que.	**121**	**122**	**123**	**124**	**125**	**126**	**127**	**128**	**129**	**130**	**131**	**132**	**133**	**134**	**135**
Ans.	3	2	3	2	1	3	3	4	3	4	1	2	3	2	4
Que.	**136**	**137**	**138**	**139**	**140**	**141**	**142**	**143**	**144**	**145**	**146**	**147**	**148**		
Ans.	2	4	1	2	3	2	1	1	3	4	3	4	1		

PREVIOUS YEAR QUESTIONS

Que.	1	2	3	4
Ans.	1	2	3	2

QUOTATION FOR STUDY MATERIAL PACKAGE

Study Material	No. Of Module	Rate Per Set (Above 50 Student)	Rate Per Set (Above 100 Student)
Class 6^{th}	6	1600/Set	1400/Set
Class 7^{th}	7	1800/Set	1600/Set
Class 8^{th}	6	2000/Set	1800/Set
Class 9^{th}	8	2200/Set	2000/Set
Class 10^{th}	9	2600/Set	2400/Set
Class 11^{th} (One Year)	NEET - 13 JEE - 12	2800/Set	2500/Set
Class 12^{th} (One Year)	NEET - 11 JEE - 11	2800/Set	2500/Set

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